Visualisation in Multidimensional Space: case of Robust Design

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Summary

1. **Basic formulation of Robust Design**
2. **Application in 2D airfoil robust design optimisation**
3. **Visualisation in Multi-D: Self Organising Maps**

Robust Design Concept

In many real industrial optimisation, the input design parameters are not fix:

Uncertainties could characterise some geometric entities (lengths, relative positions, angles,etc.);

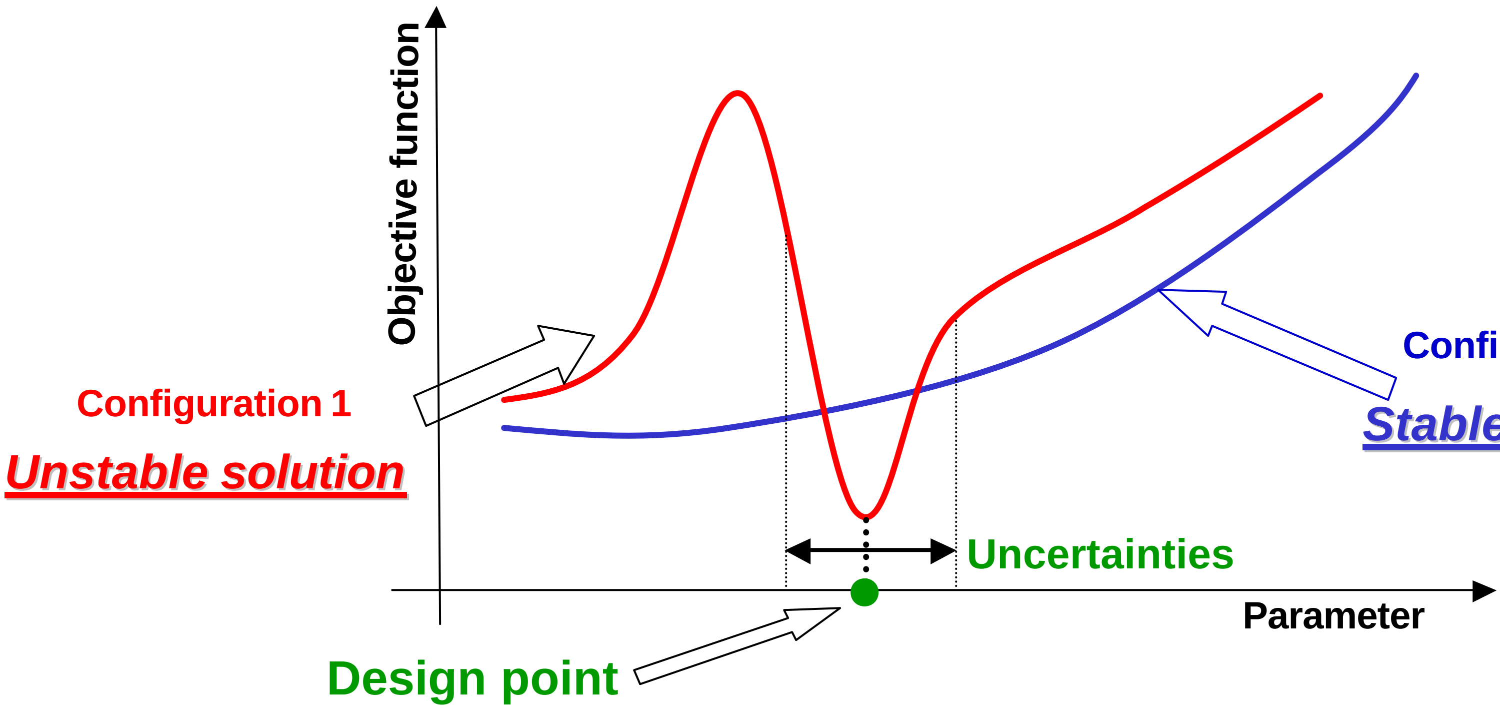
The operative conditions are not fixed cause of fluctuations

Turbomachines: the mass flow rate, inlet pressure, etc.

Aeronautic: flight speed, angle of attach, etc.

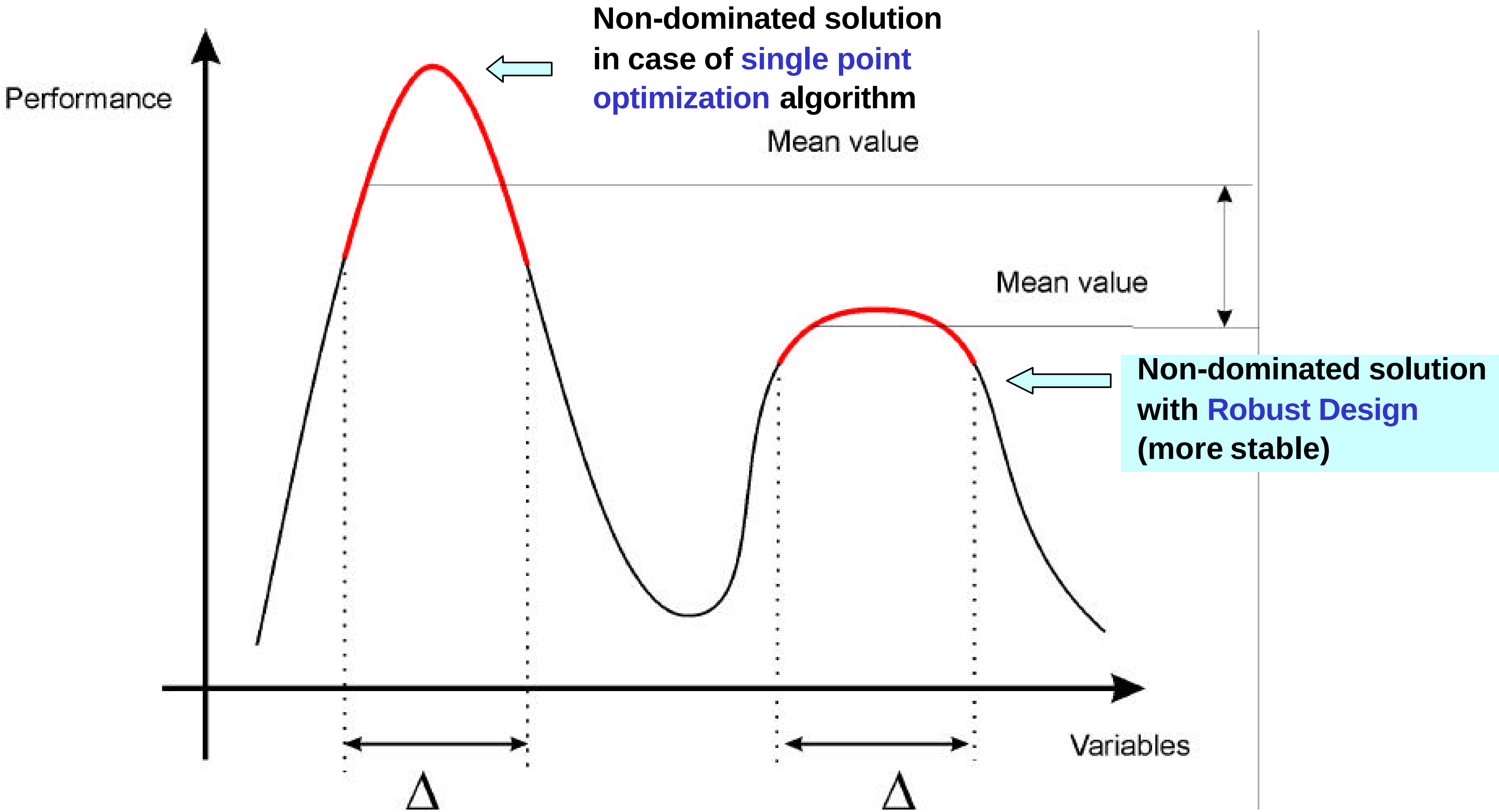
Robust Design Concept

- **When there is the presence of fluctuations of the operating conditions it is important to define the stability of one solution;**
- **Traditional optimization techniques tend to "over-optimize", producing solutions that perform well at the design point but have poor off-design characteristics.**



ROBUST DESIGN

- **What happens when we optimise a function in which the input design parameters are defined by the mean value (Xm) and the deviation ?? ???**



ROBUST DESIGN

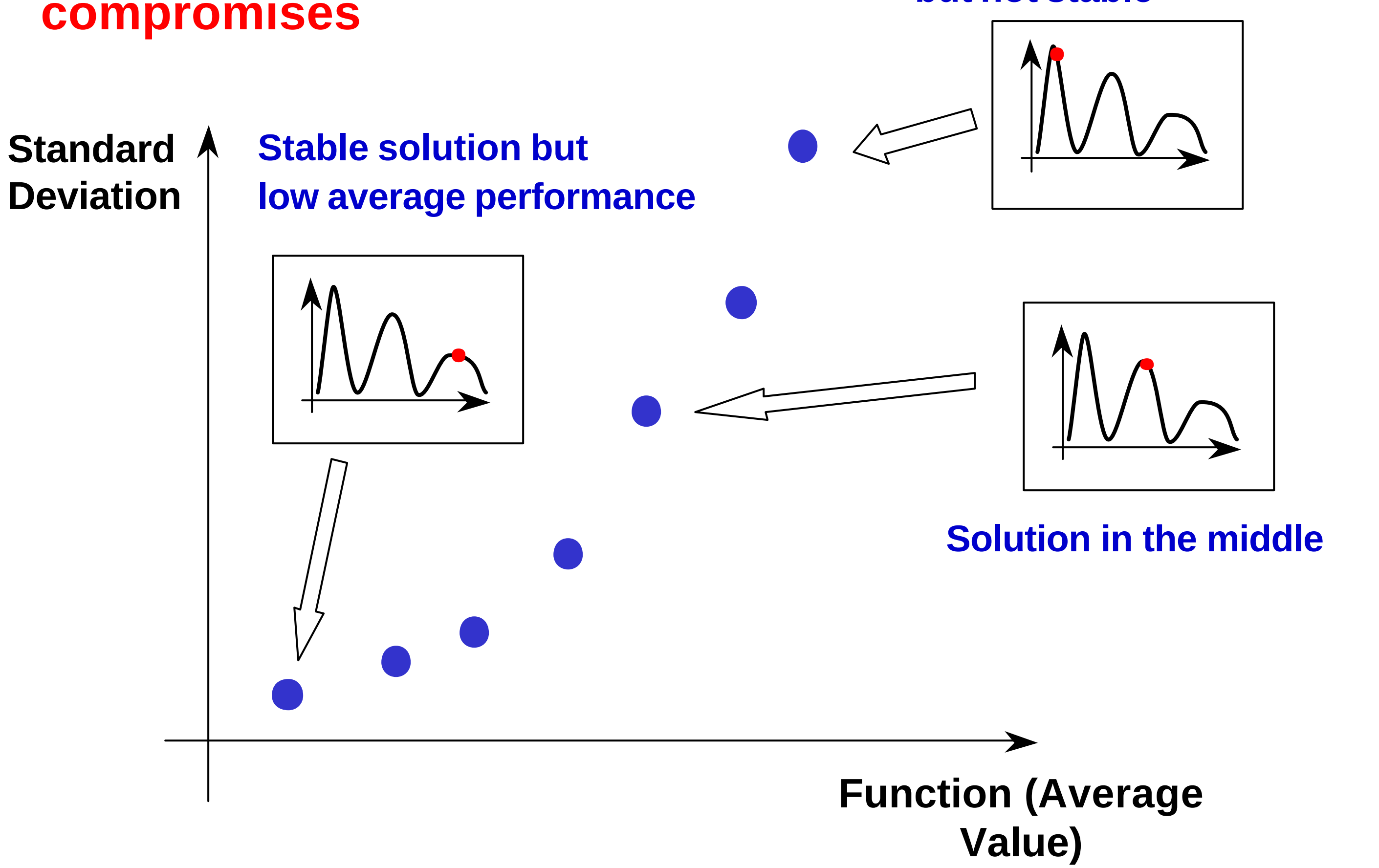
So when there is the presence of fluctuations a Multi Objective Approach is needed

- Maximise the mean value of the function (performance)
- Minimise the variance of the function (stability)

ROBUST DESIGN

What do we need?

We need the best compromises



ROBUST DESIGN

Mathematematic formulation of the objective functions

$$\max f(x_i, q_i) \quad i = 1,..,k,..,n$$

$p(q_k)$ probability distribution of uncertainties q_k

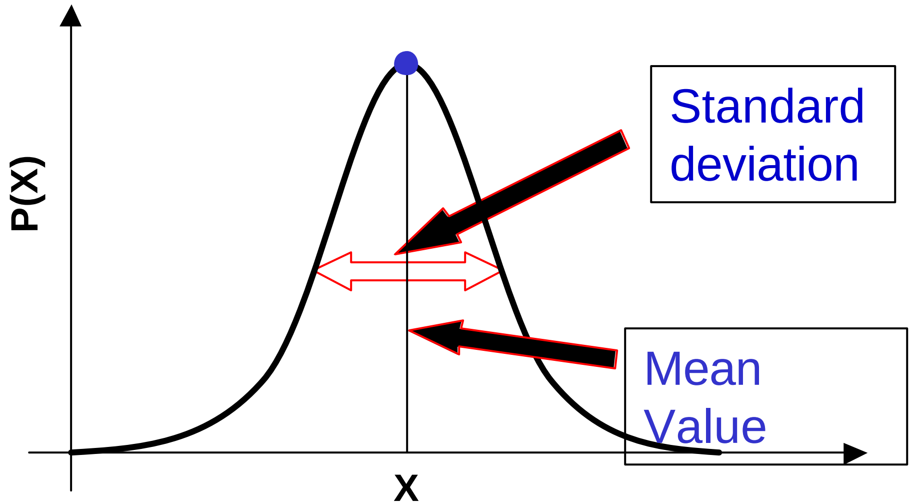
with **Robust Design Theory** becomes:

$\max E(f_i) = \int f_i(x,u)\,p(u)\,du$	Mean value
$\min \sigma^2(f_i) = \int \left[f_i(x,u) - E(f_i)\right]^2 p(u)\,du$	Variance

ROBUST DESIGN

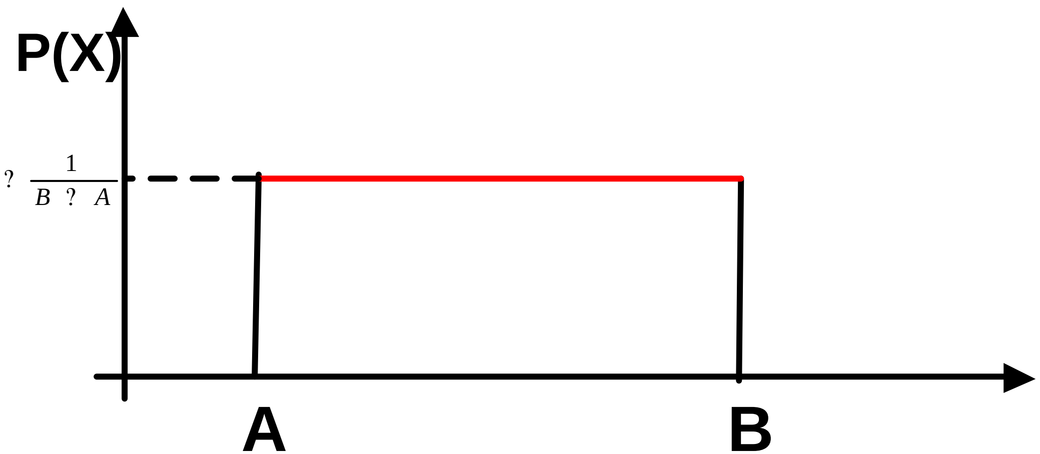
It is possible to use different probabiliy density functions:

Gaussian



Uniform

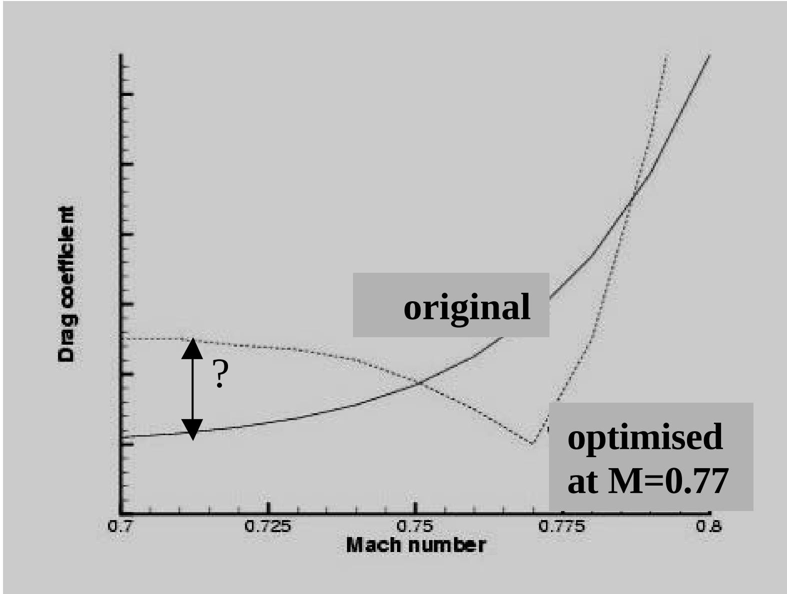
$P(x) ? \frac{1}{B ? A}$



Application in robust design airfoil optimization

- It is possible to illustrate the concept of Robust Design considering a 2D airfoil shape optimization problem in transonic field.

It has been observed (Hicks and Vanderplaats, 1977) that minimizing drag at a single design point causes reduction of performances (?) at nearby off-design points.



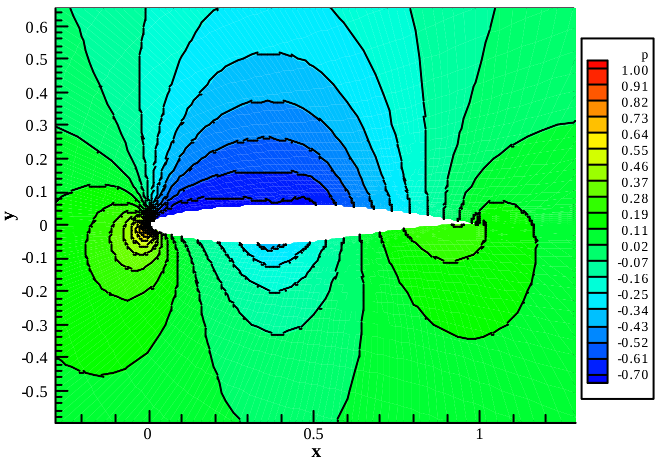
Thus, it is necassary to optimise drag with (two) input parameters given by mean values and deviation

MACH=0.73?0.05   ? =2^o ?0.5

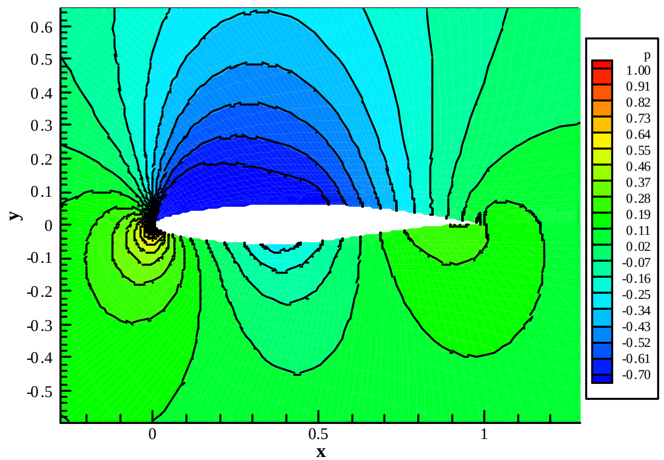
Uniform density function

Mach Field

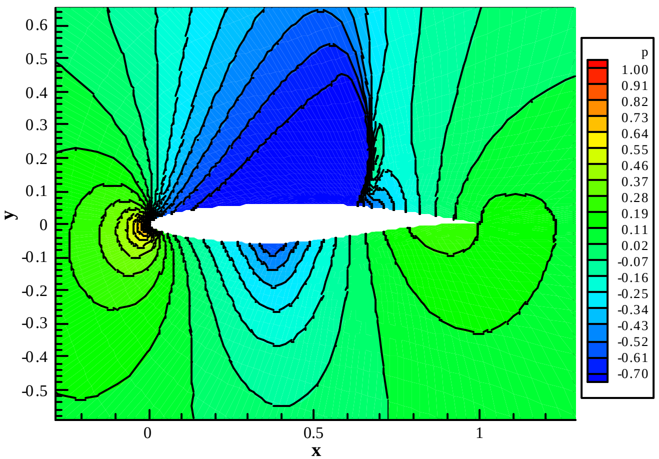
Mach 0.68, Alpha 1.5



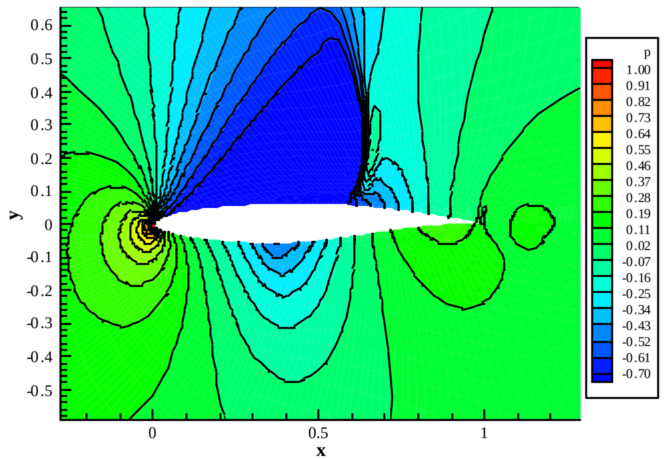
Mach 0.68, Alpha 2.5



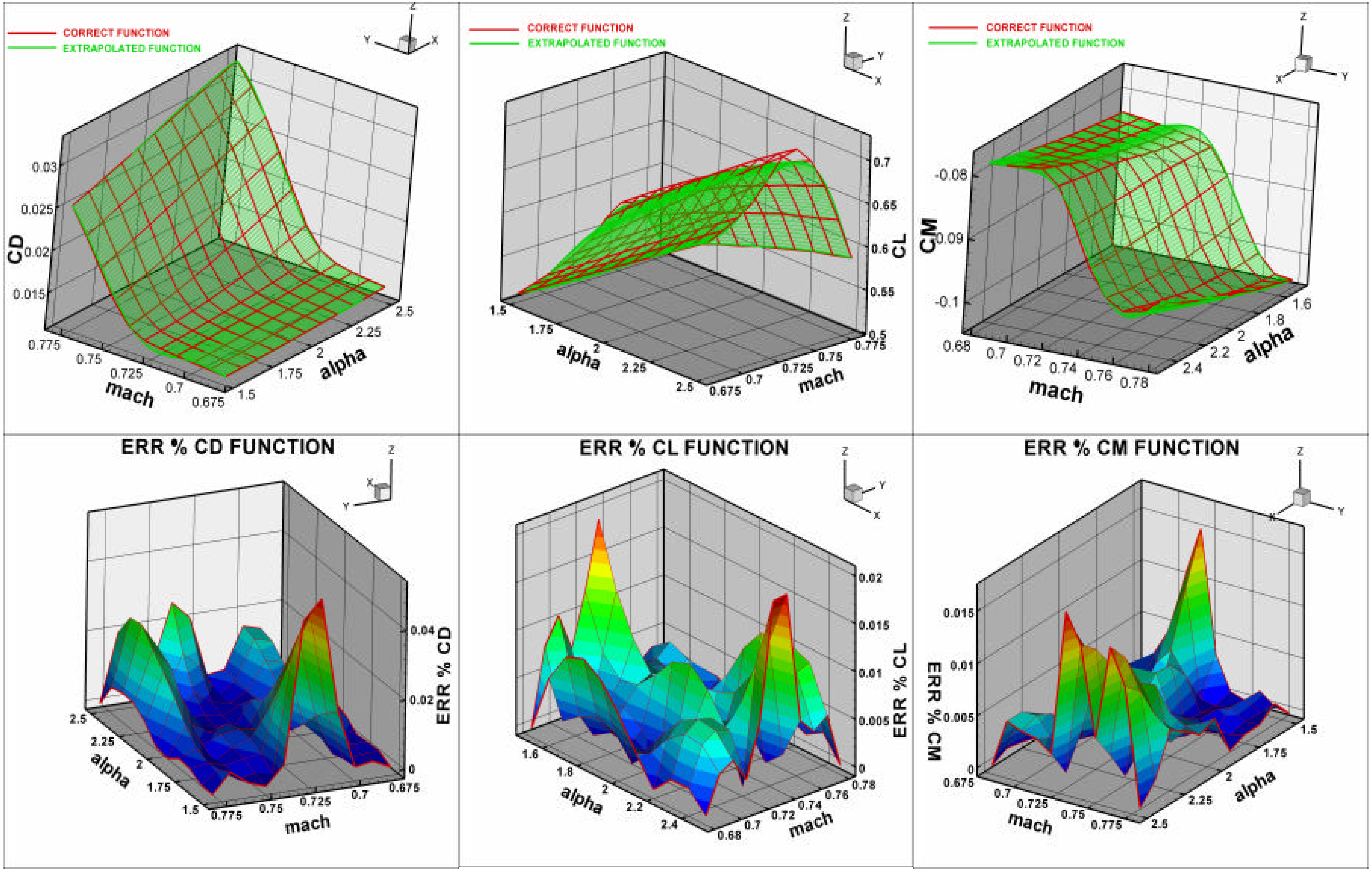
Mach 0.78, Alpha 1.5



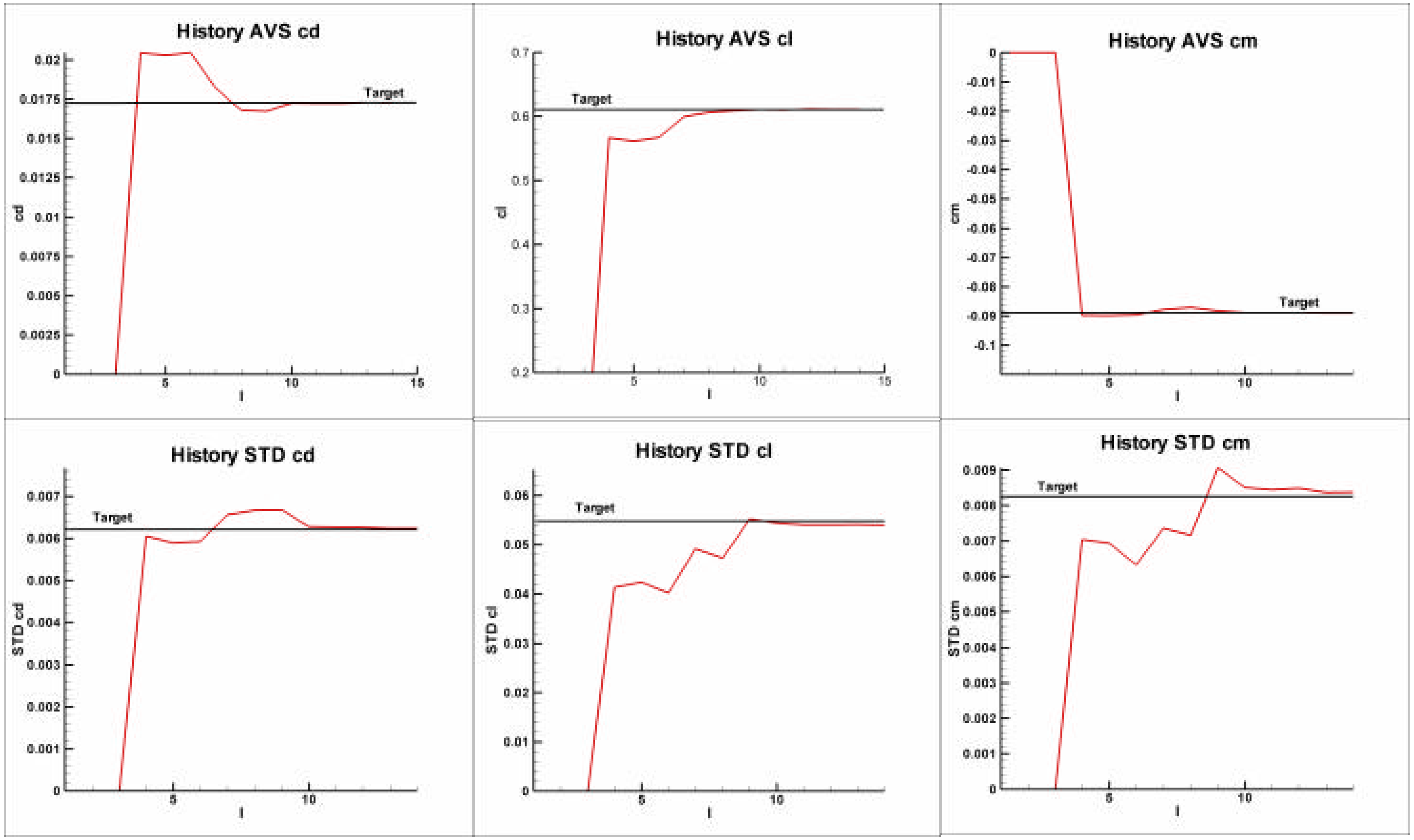
Mach 0.78, Alpha 2.5



REAL SURFACES Cl, Cd,Cm



Convergence

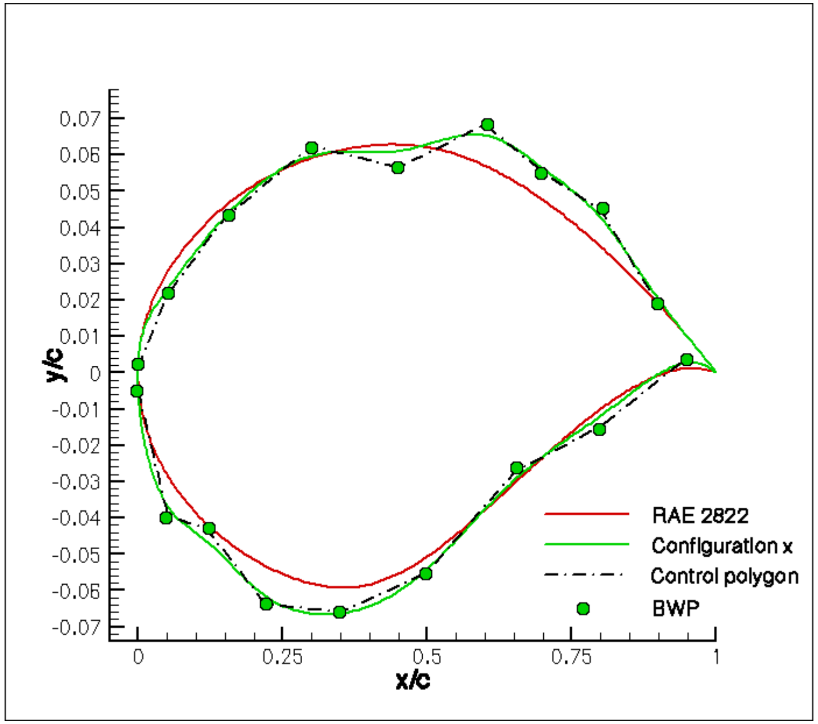


10 iterations < 5% error

Parameterization and baseline configuration

- In the optimization process the achievable configurations have been determined by modifying an baseline configuration, the supercritical airfoil RAE 2822 designed by the Royal Aircraft Establishment

- Parameterisation of the airfoil by means 18 design variables (Bezièr weighting points).



TEST CASE: RAE2822

This problem of Robust Design can be expressed as:

$$\left.\begin{array}{ll} \max & E(c_l) \\ \min & ?^2(c_l) \end{array}\right\} \quad \text{Lift}$$

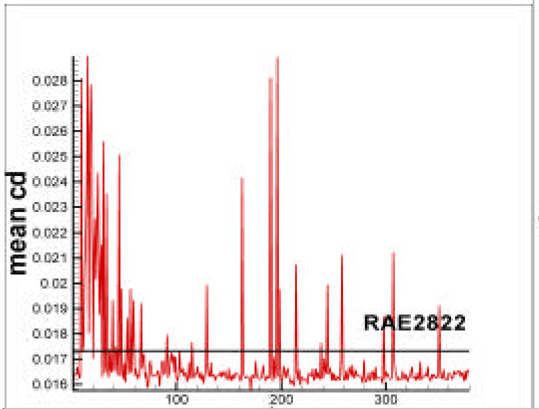
$$\left.\begin{array}{ll} \min & E(c_d) \\ \min & ?^2(c_d) \end{array}\right\} \quad \text{Drag}$$

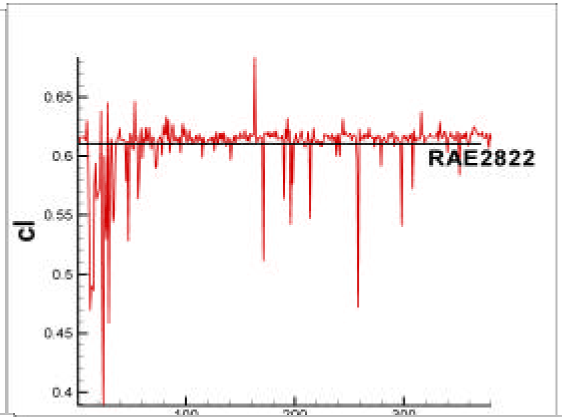
with constraints:

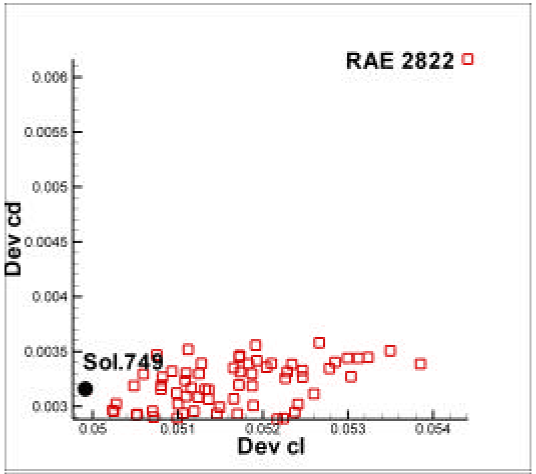
$$\left\{\begin{array}{l} E(c_l)\ ?\ E(c_l)^* \\ E(c_d)\ ?\ E(c_d)^* \\ E(c_m)\ ?\ E(c_m)^* \end{array}\right. \qquad \left\{\begin{array}{l} ?^2(c_l)\ ?\ ?^2(c_l)^* \\ ?^2(c_d)\ ?\ ?^2(c_d)^* \\ ?^2(c_m)\ ?\ ?^2(c_m)^* \end{array}\right.$$

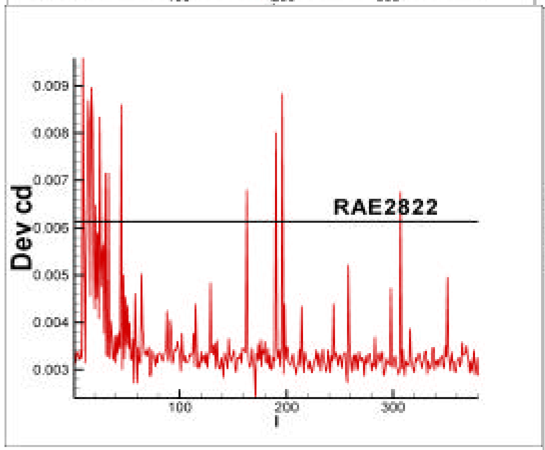
where $E(c_l)^*, E(c_d)^*, E(c_m)^*, ?^2(c_l)^*, ?^2(c_d)^*, ?^2(c_m)^*$
are respectively the mean values and the variances of the original airfoil (RAE2822).

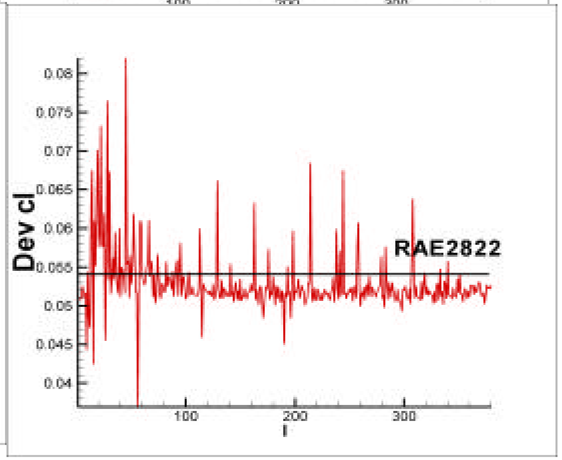
TEST CASE: RAE2822

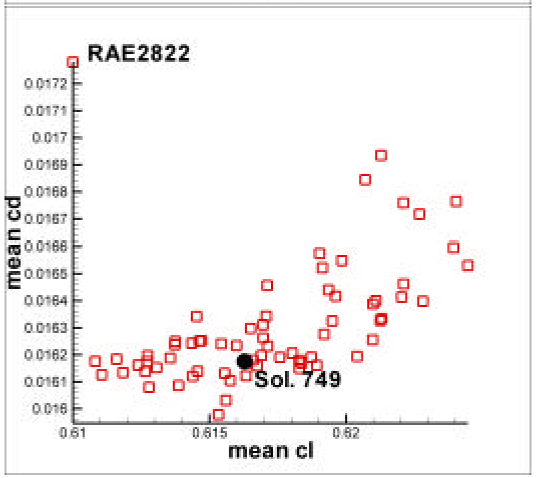










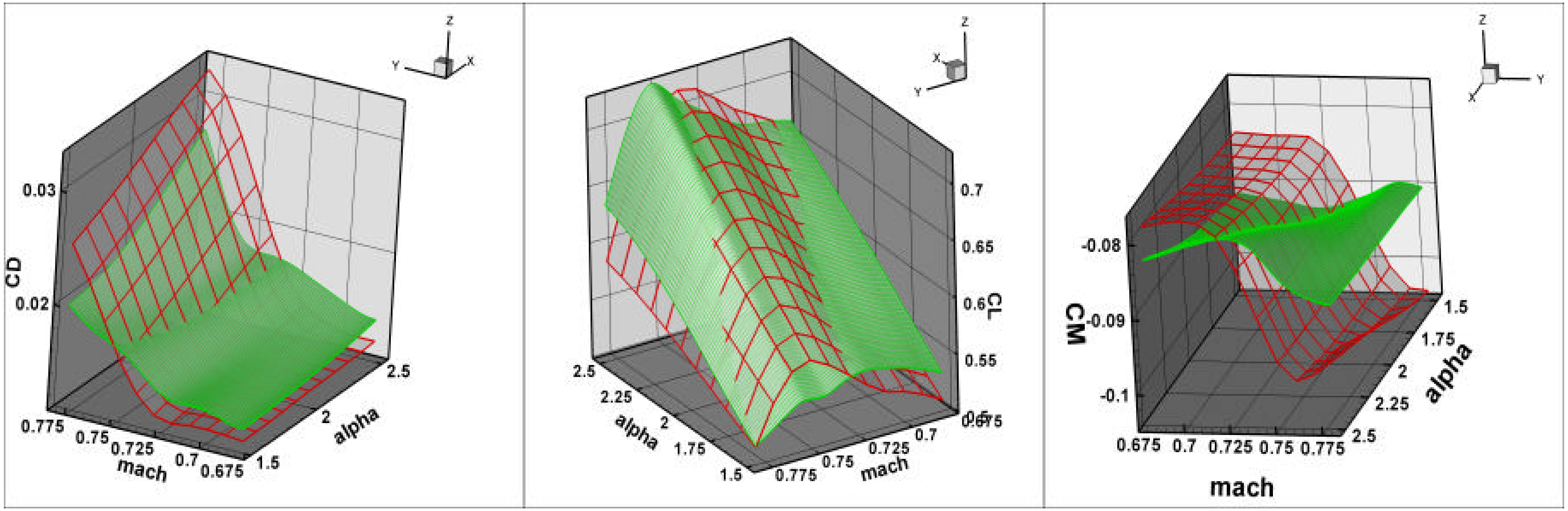


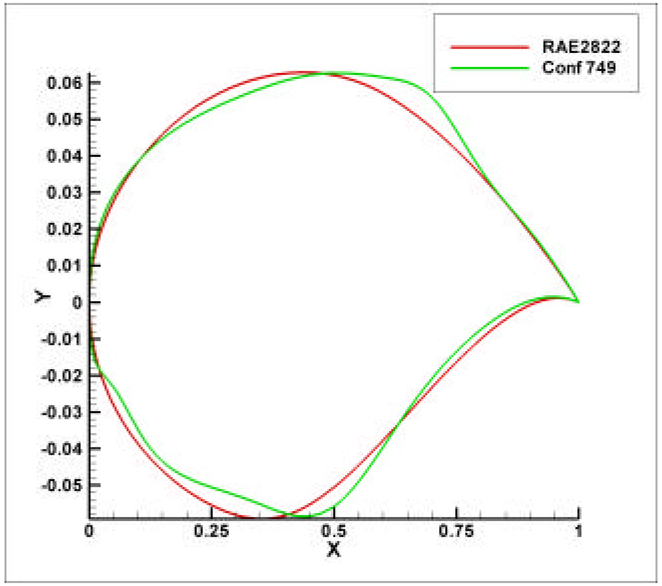
- **Using a MCDM methodology with this weights:**

E(cl)=6; E(cd)=10; Dev(cl)=2; Dev(cd)=8

The best solution will be the 749

RAE2822 (Results)





Obiettivi	RAE2822	Sol. 749
E(Cl)	0,61	0,616
E(Cd)	0,017	0,01616
Dev(Cl)	0,05441	0.04976
Dev(Cd)	0,06164	0,03434

Visualization in Multi-D (SOM)

Clustering of data ? **Self-Organizing Maps**

SOM is a two-dimensional array of neurons

$M=\{m_1,\ldots\ldots,m_{pxq}\}$ m same dimension as the input vectors

(objectives)

Best-matching unit:

$$\|x \; ? \; m_c\| \; ? \; \min_i \; ?\|x \; ? \; m_i\|?$$

Algorithm SOM – Basic Idea

- T. Kohonen* in the ’80
- The basic idea is the capacity of the brain to organize in topographical way the data; every stimulus is proceeded in specific zones.
- The numerical algorithms which use this basic idea are called *self-organizing*; the Self Organizing Maps are the main example.
- The SOM performs the association between the output and ordered units (in 2D space); so it is possible to have an ordered view of the data.

Algorithm SOM

1) $\mathbf{X}^T=(X_1, X_2 .. X_d)$, data to be rapresented.

Build a grid with i nodes ($i=1,\ldots,k$) in 2D. Every node get a d-dimensional vector $\mathbf{W}^{(i)T} = (W_1^{(i)}\ W_2^{(i)} .. W_d^{(i)})$, $\mathbf{W}^{(i)} = \mathbf{W}^{(i)}(t)$ (*codebook*), changes iteractively.

2) Initialization: random values $\mathbf{W}^{(i)}(0)$ for the nodes $i=1...K$.

Initial parameters *neighborhood function*

$$?(t)h(|r_i ? r_c|/?(t),t)$$

3) Selection of one $\mathbf{X}^T$

Algorithm SOM

3.1) Distance $d(\mathbf{X},\mathbf{W}^{(i)})$, find $\mathbf{W}^{(c)}$ similar to (closer to) $\mathbf{X}$

3.2) Update of every component values in the $O(r_c)$

3.3) Decrease $h_0(t)$ and $?(t)$.

3.4) End of the iterations

4) Quantisation errors (quality of the maps) $E_q ?\mathbf{W}?? \; ? \, ?_{c?1}^{m} \, ?_c \, ?\mathbf{W}?^2 \, ?^{1/2}$

Utility of SOM

- Visualisation of data n-dimensional
- Exploration of data
- Classification

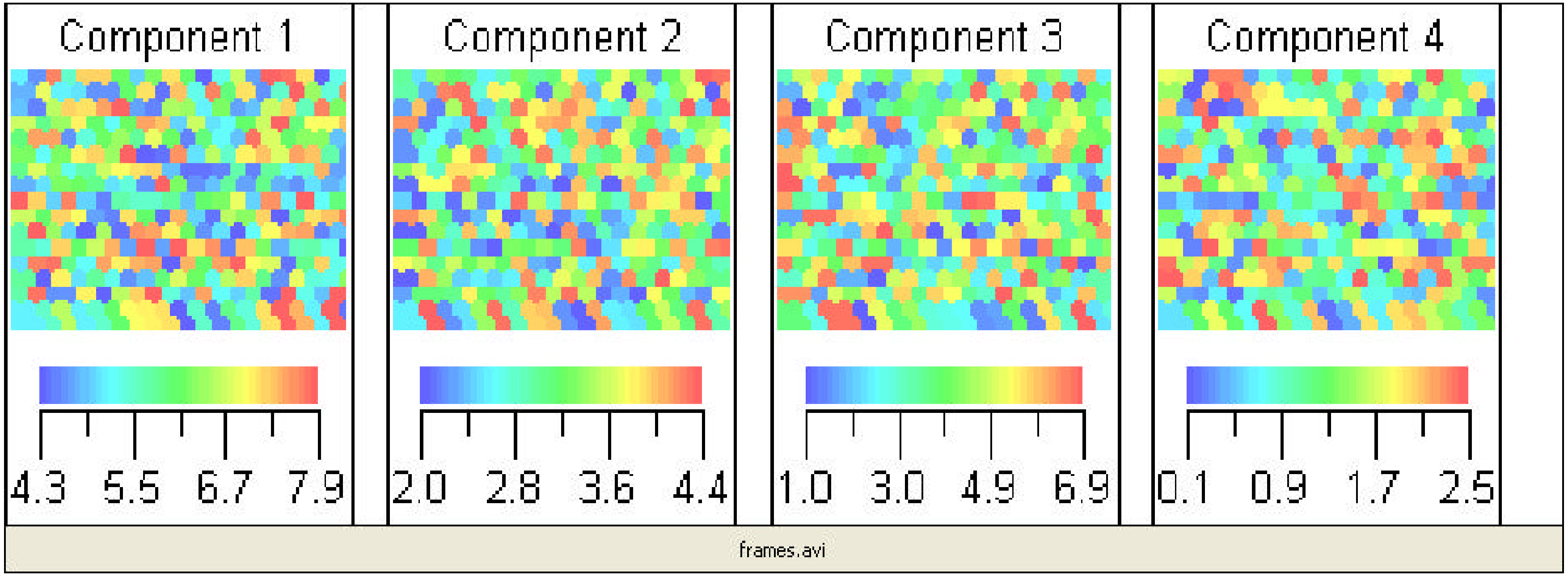
Utility of SOM- Iris - Data

SepalL	SepalW	PetalL	PetalW	Type
4.6	3.6	1.0	0.2	Setosa
5.1	3.3	1.7	0.5	Setosa
4.8	3.4	1.9	0.2	Setosa
5.0	3.0	1.6	0.2	Setosa
5.0	3.4	1.6	0.4	Setosa
6.5	2.8	4.6	1.5	Versicolor
5.7	2.8	4.5	1.3	Versicolor
6.3	3.3	4.7	1.6	Versicolor
4.9	2.4	3.3	1.0	Versicolor
6.6	2.9	4.6	1.3	Versicolor
7.6	3.0	6.6	2.1	Virginica
4.9	2.5	4.5	1.7	Virginica
7.3	2.9	6.3	1.8	Virginica
6.7	2.5	5.8	1.8	Virginica
7.2	3.6	6.1	2.5	Virginica
6.5	3.2	5.1	2.0	Virginica

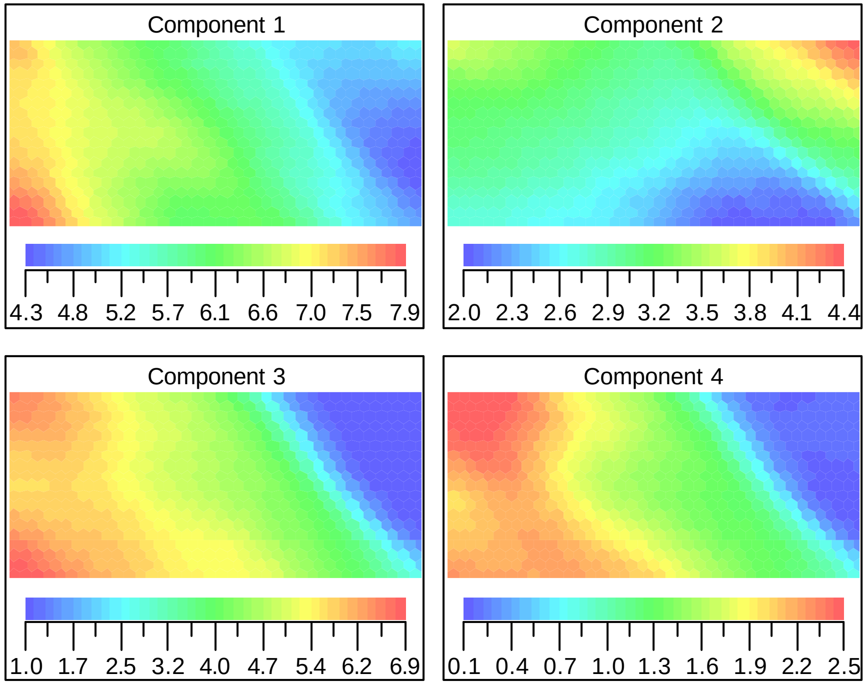
150 data, 4 variables

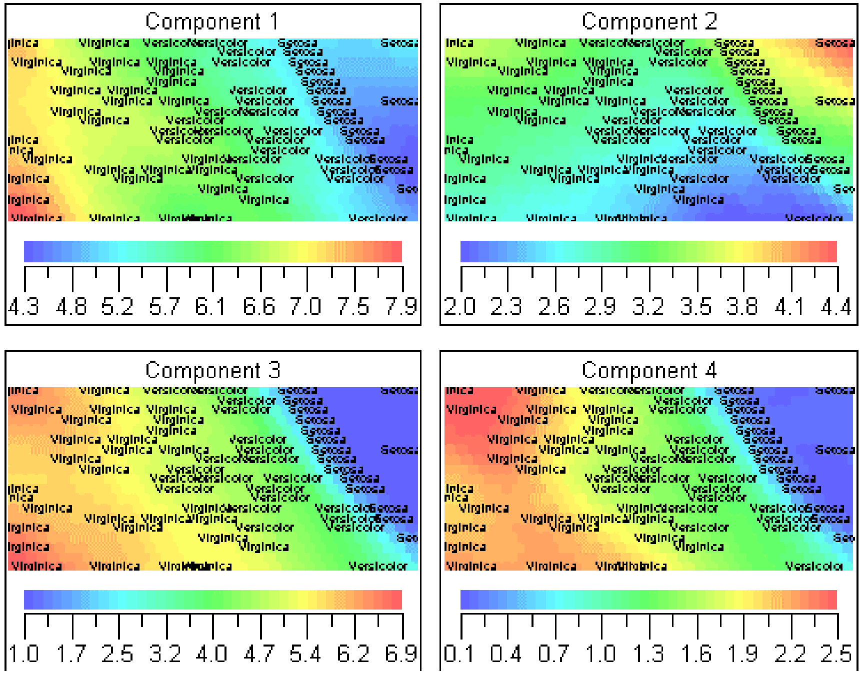
3 typology of Iris

Algorithm SOM - Dynamic



IRIS EXAMPLE



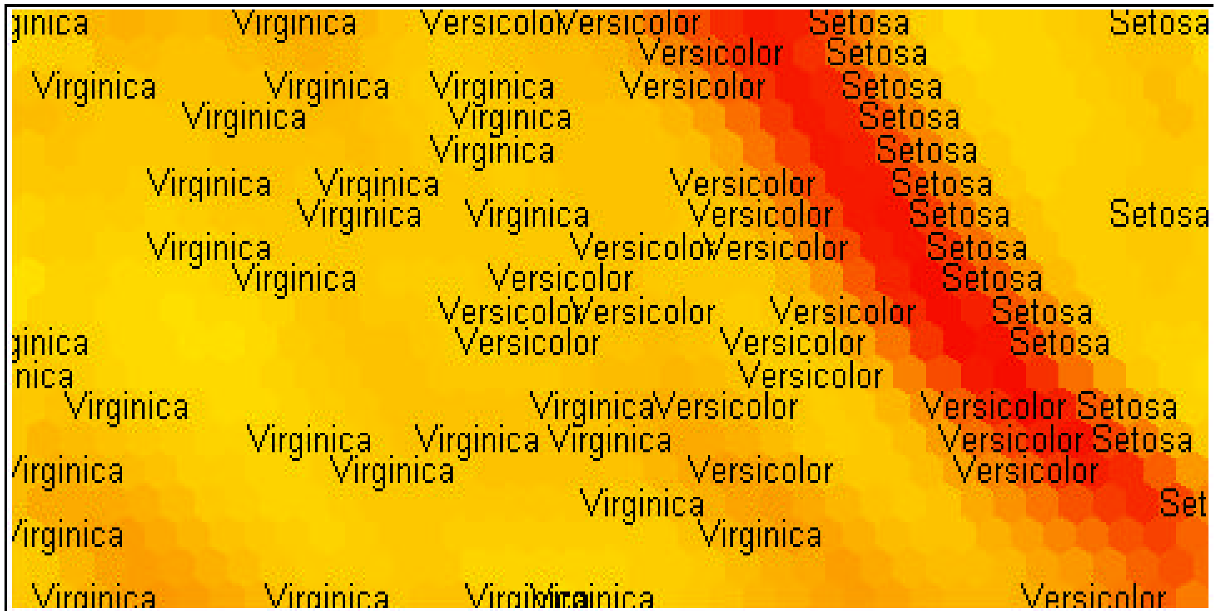


Local correlation
non linear,

Every data is a node

Clustering

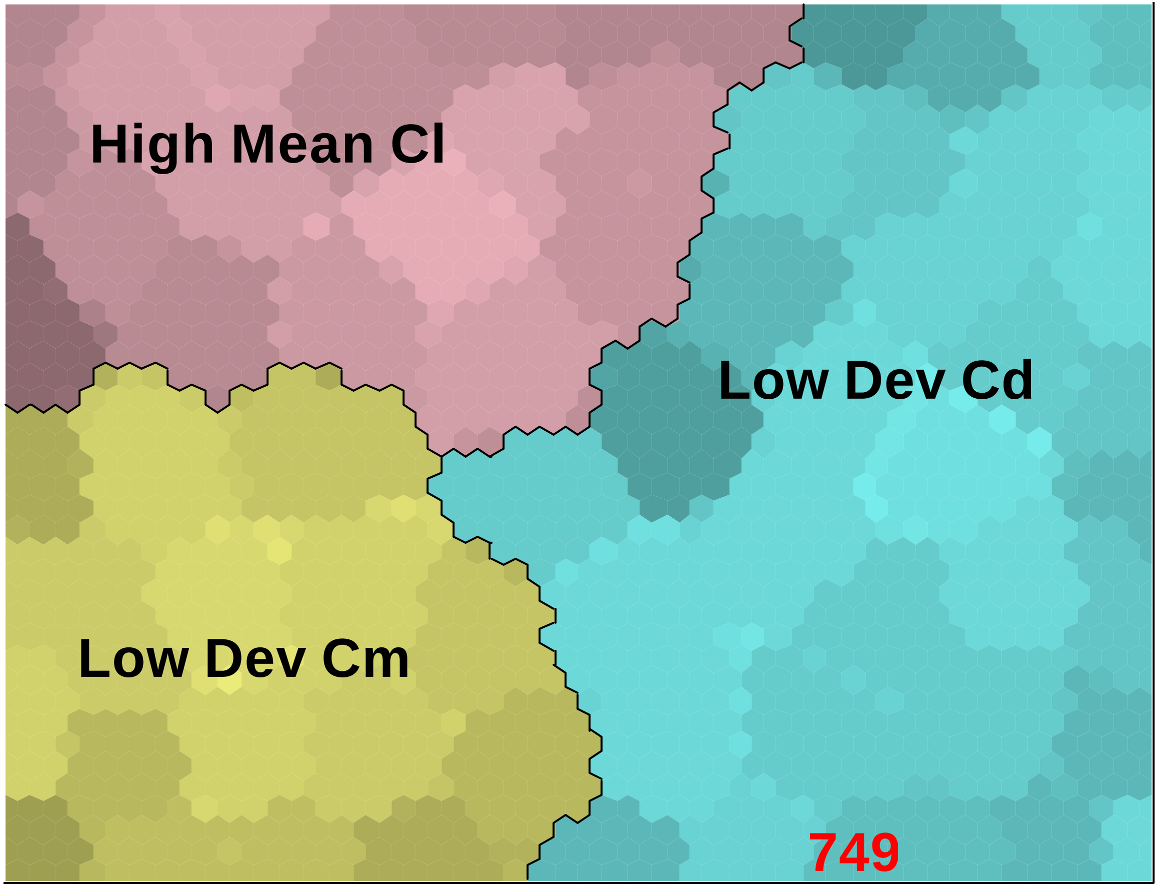
U-Matrix



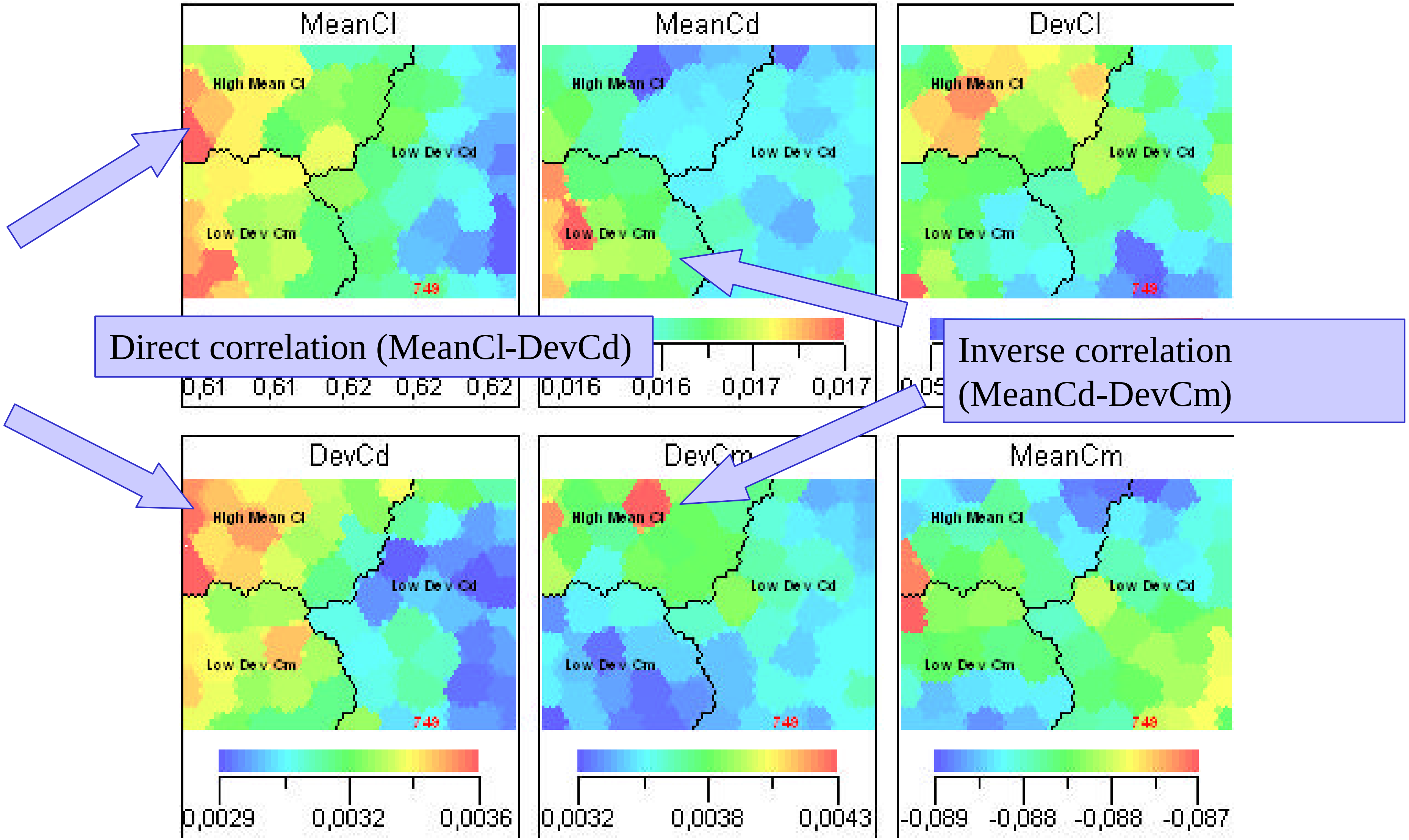
Clustering no supervised: with the U-matrix, based on the distance Between the data, it is possible to rappresent the *cluster* (collection of similar data)

Visualization in Multi-D (SOM)

SOM with the data of the Pareto Front (previous optimization)

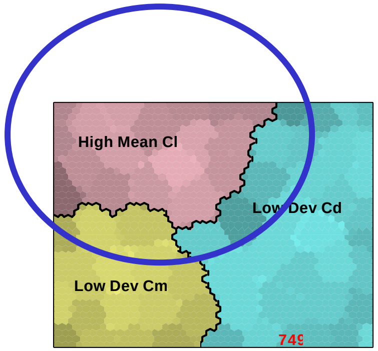


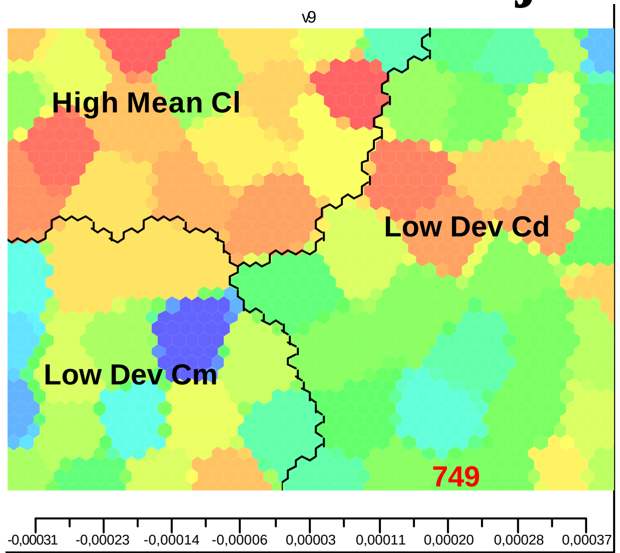
Visualization in Multi-D (SOM)



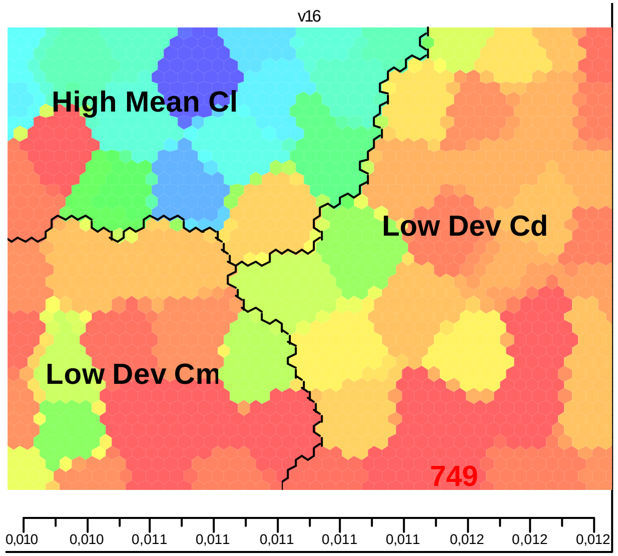
Visualization in Multi-D (SOM)

Easy visualization of the relations between objectives and variables



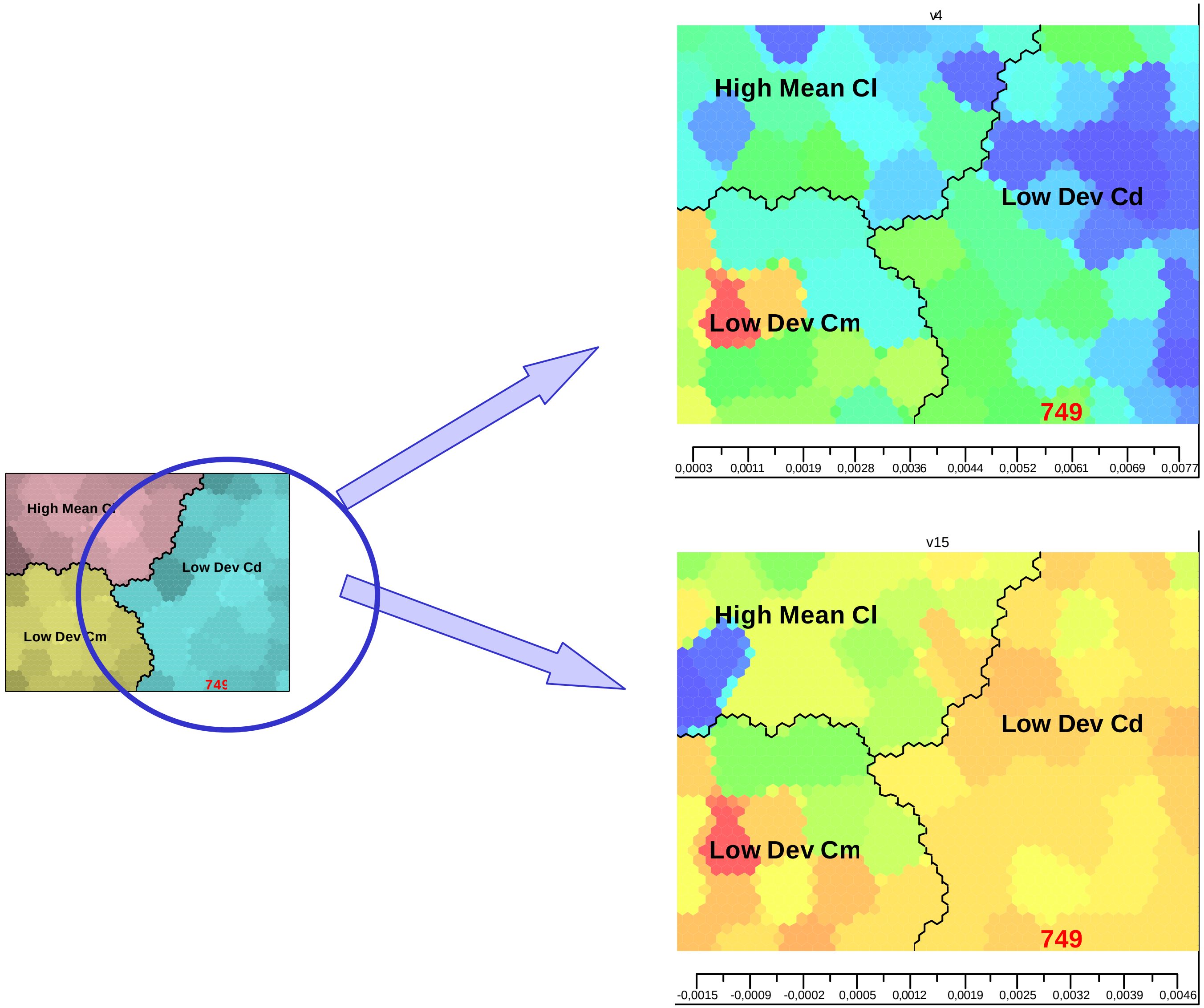


V9



V16

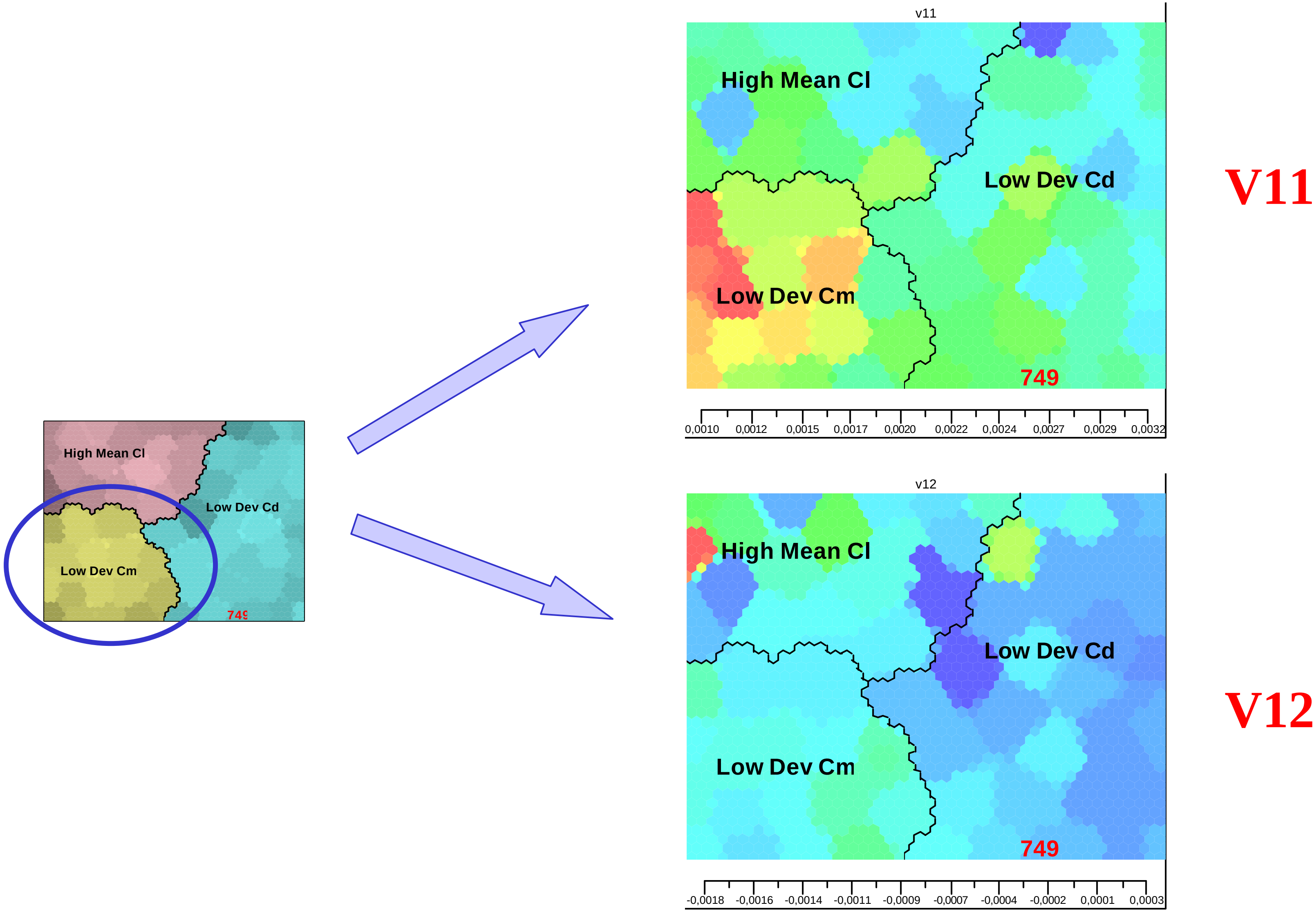
Visualization in Multi-D (SOM)



V4

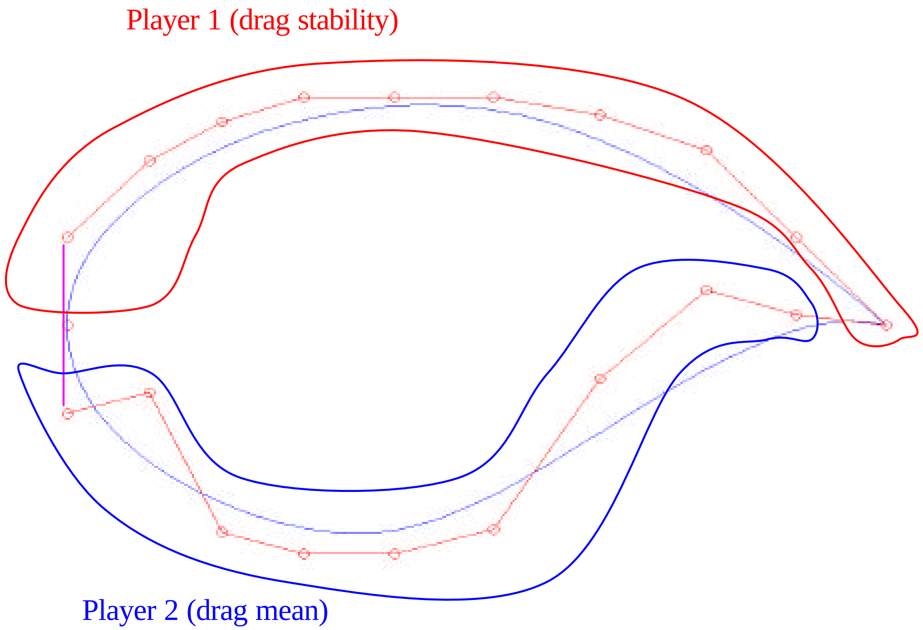
V15

Visualization in Multi-D (SOM)



Game theory approach (Nash)

Objectives: minimise Mean(C_d) and Std (C_d)



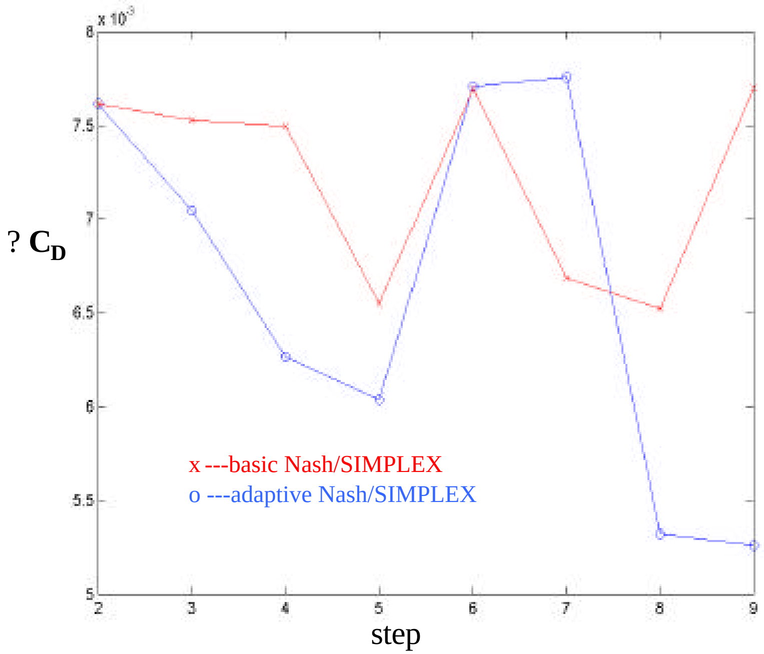
Initial variables space decomposition:

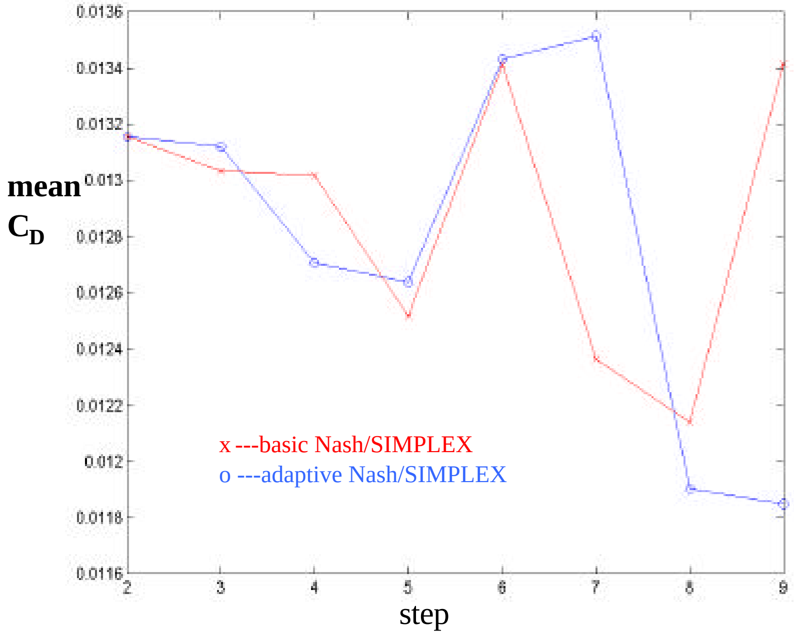
9 co-ordinates Bèzier points upper side – standard deviation of drag

9 co-ordinates Bèzier points lower side – mean value of drag

Game theory approach (Nash)

Results of optimisation: basic algorithm and adaptive one



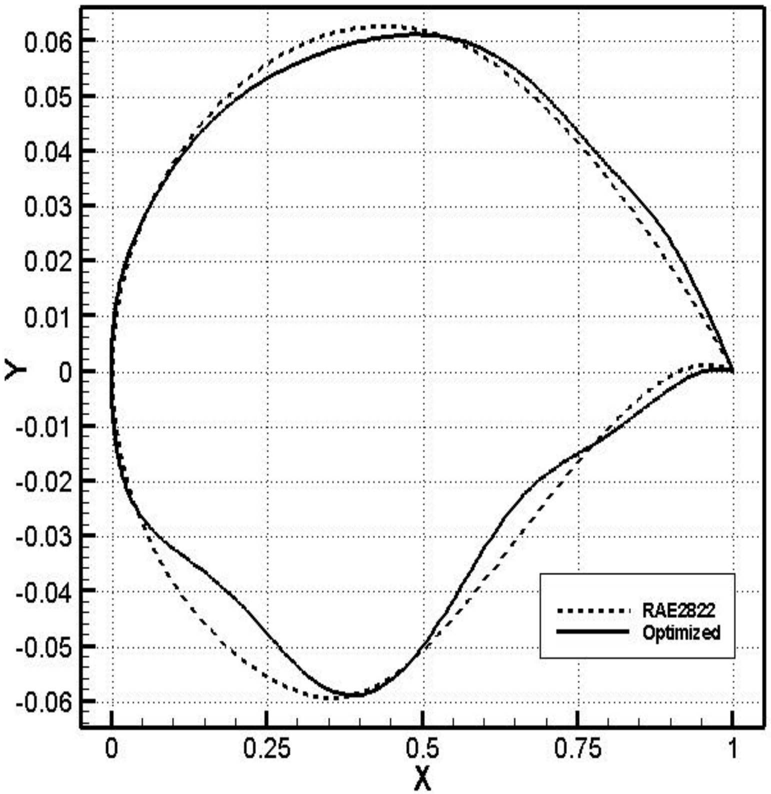


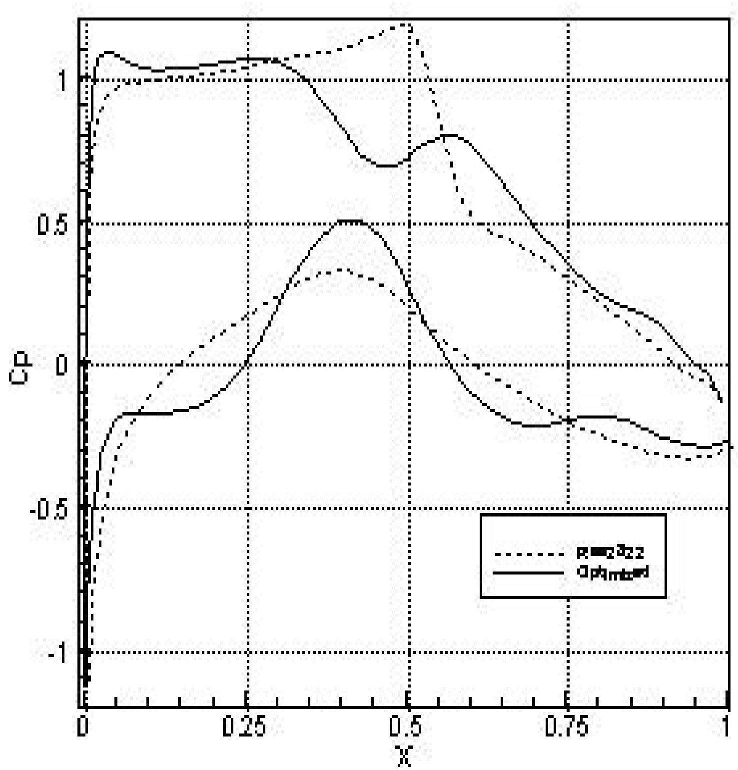
	MeanCd (obj)	**? Cd (obj)**	MeanCl (constr)	? Cl (constr)	MeanCm (constr)	? Cm (constr)
RAE2822	**1.33e-2**	**7.6e-3**	0.686	5.91e-2	-0.105	9.50e-3
BEST	**1.18e-2**	**5.27e-3**	0.691	5.83e-3	-0.102	6.45e-3

Total number of design: about 500

Game theory approach (Nash)

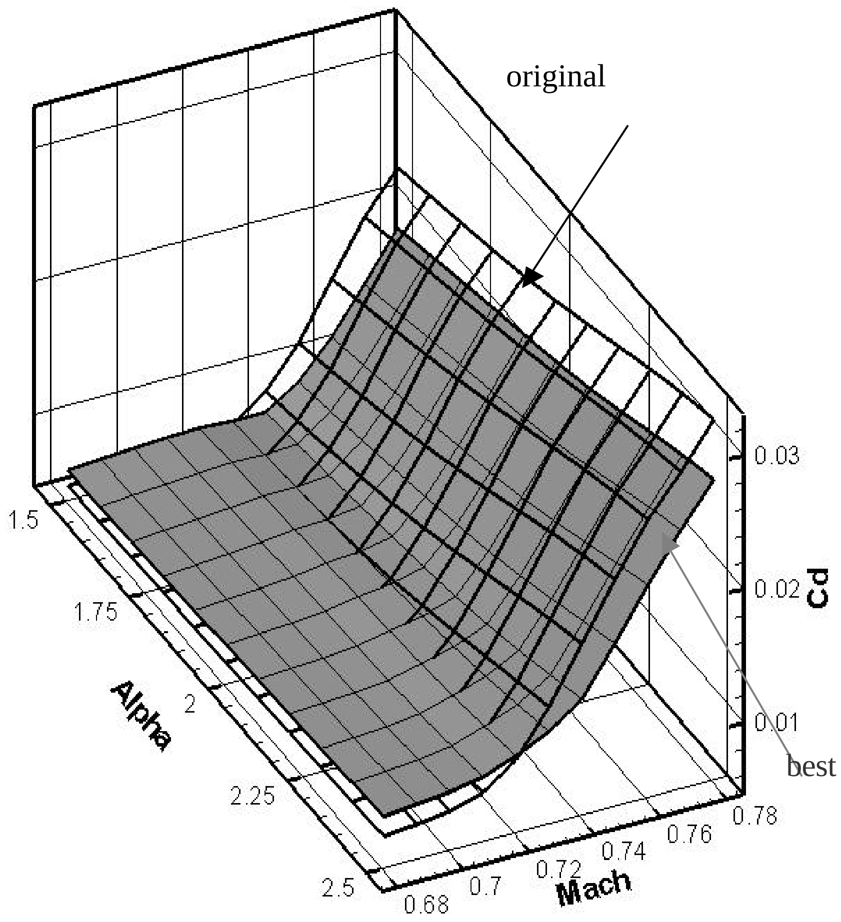
Comparison of original RAE with best configuration

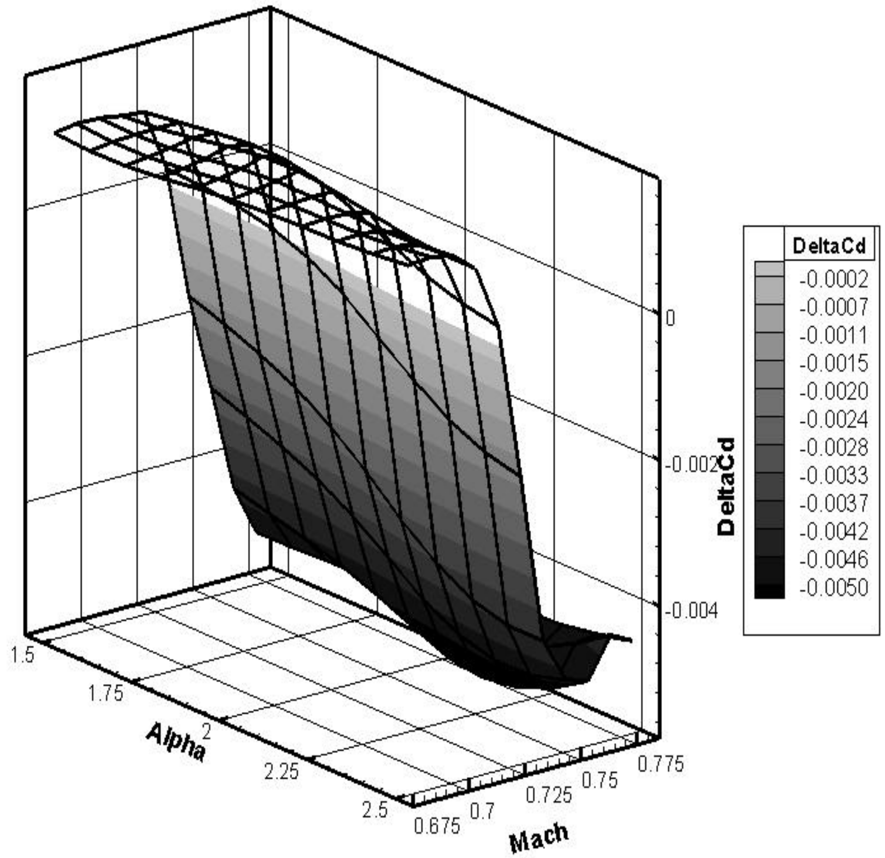




Game theory approach (Nash)

Drag surface of best configuration: difference with respect to RAE

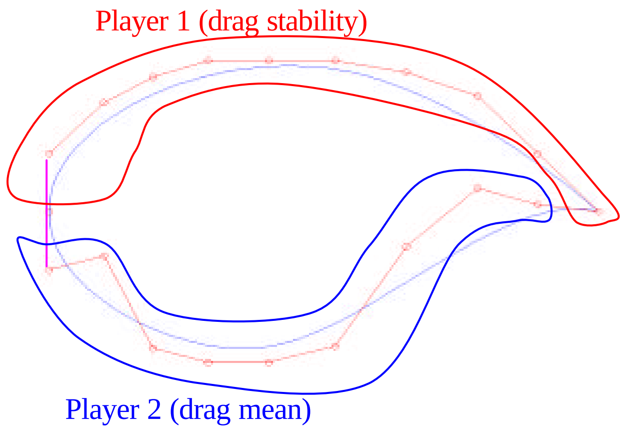




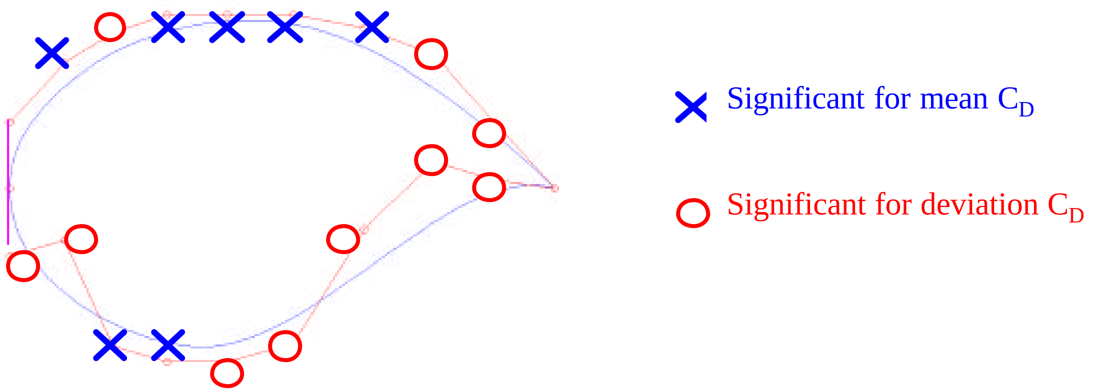
Game theory approach (Nash)

Final variables space decomposition found by adaptive Nash/SIMPLEX

Original guess



Final decomposition



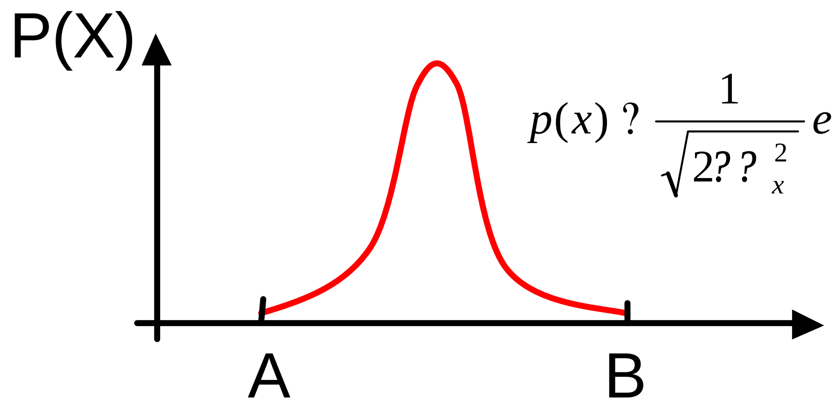
Multi Point with Gaussian probability distribution

2 design point:

SUBSONIC **? =9° (? =0.33°) Mach=0.12 (? =0.0066)**

TRANSONIC **? =2° (? =0.16°) Mach=0.73 (? =0.01)**

Gaussian probability distribution of uncertanties



$$p(x)\ ?\ \frac{1}{\sqrt{2??\,_x^2}}\,e^{?\frac{1}{2}\frac{(x?\bar{x})^2}{?\,_x^2}}$$

Gaussian distribution

Multi Point with Gaussian probability distribution

4 Objectives

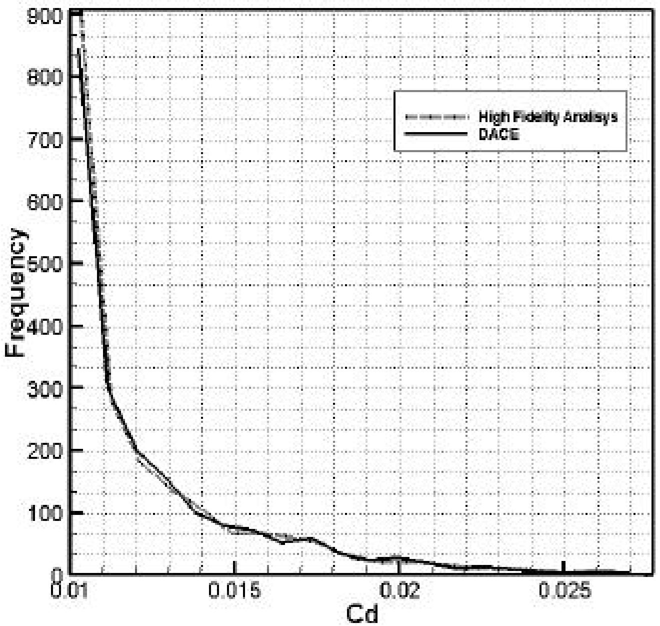
Subsonic	**Transonic**
Max Mean Efficiency	Min Mean Cd
Min Dev Efficiency	Min Dev Cd

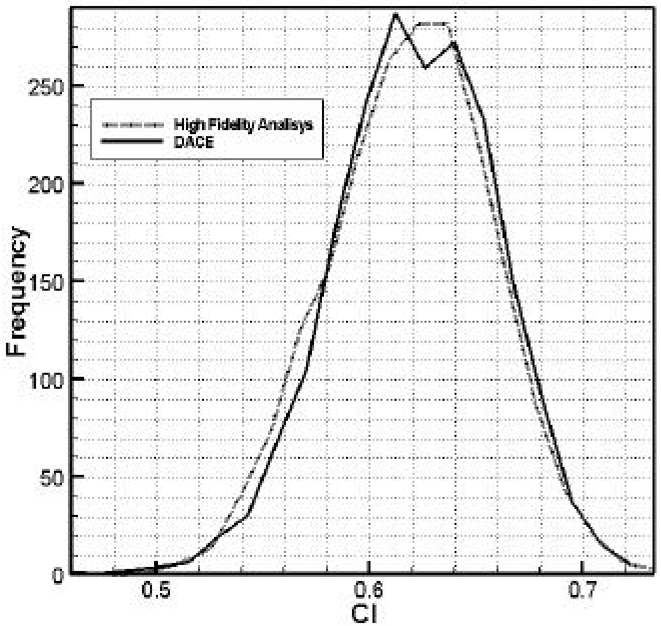
Constraints performances better that the RAE (thikness too)

DACE

CFD code: Euler + Boundary layer (Drela code, Mses)

Very interesting results with Adaptive Dace Approach



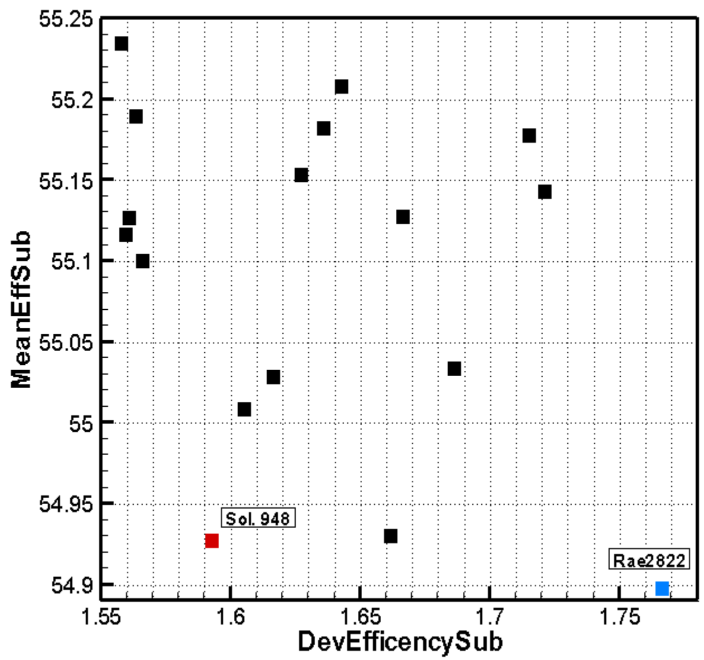


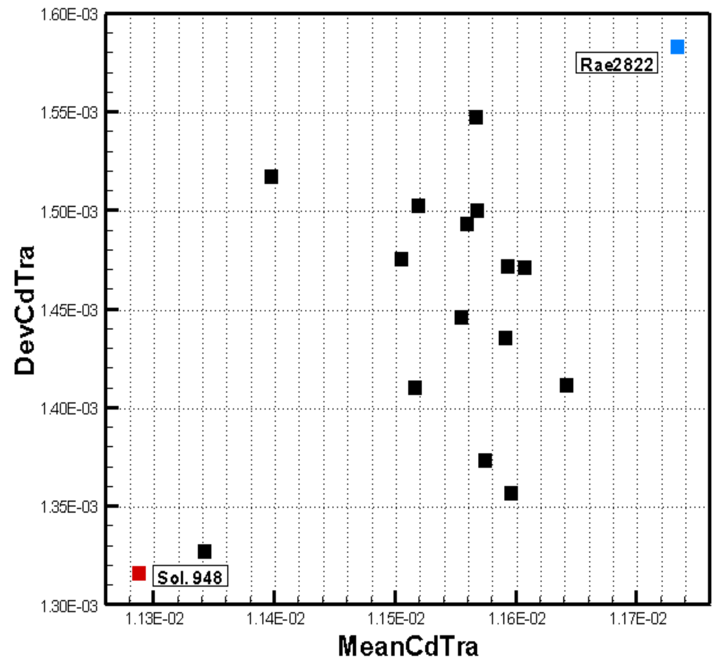
Optimization Approach

A MOGA has been used (60 ind, 16 gen.)

CODASID for determining the best in the Pareto Front

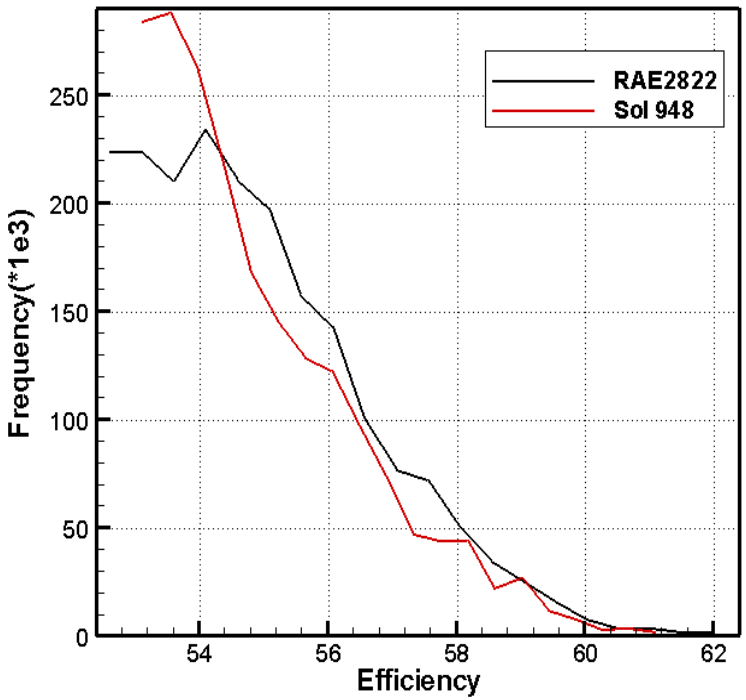
(MeanEff=10, DevEff=6, MeanCd=10,DevCd=8)

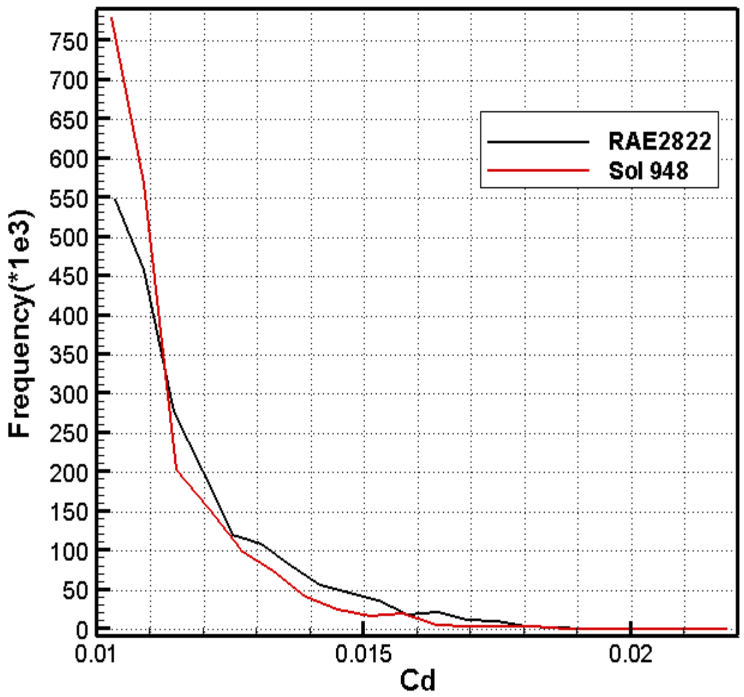




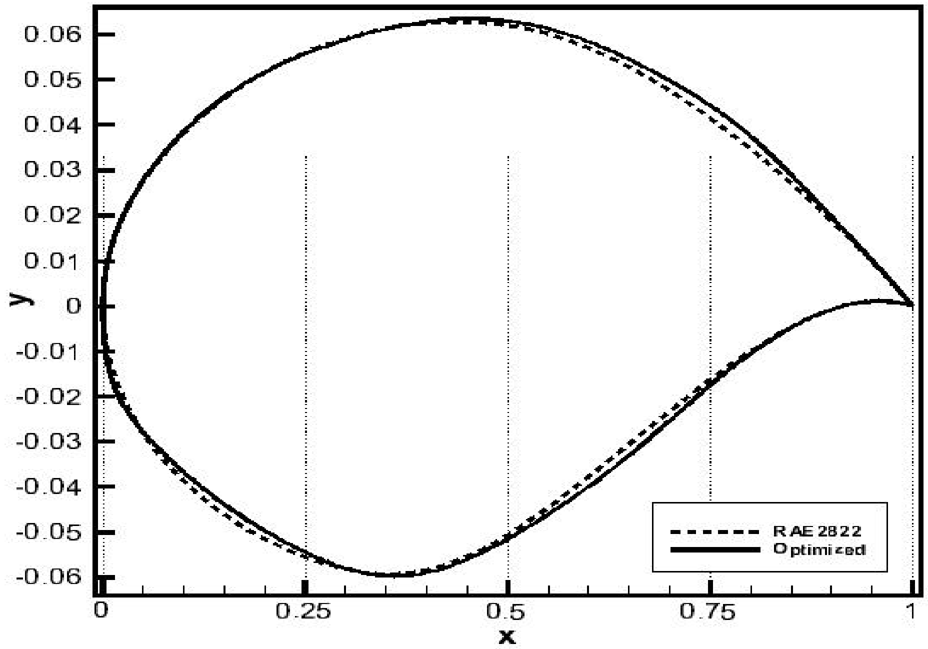
RESULTS

More importance to drag (Mean and Std)





AIRFOIL COMPARISON



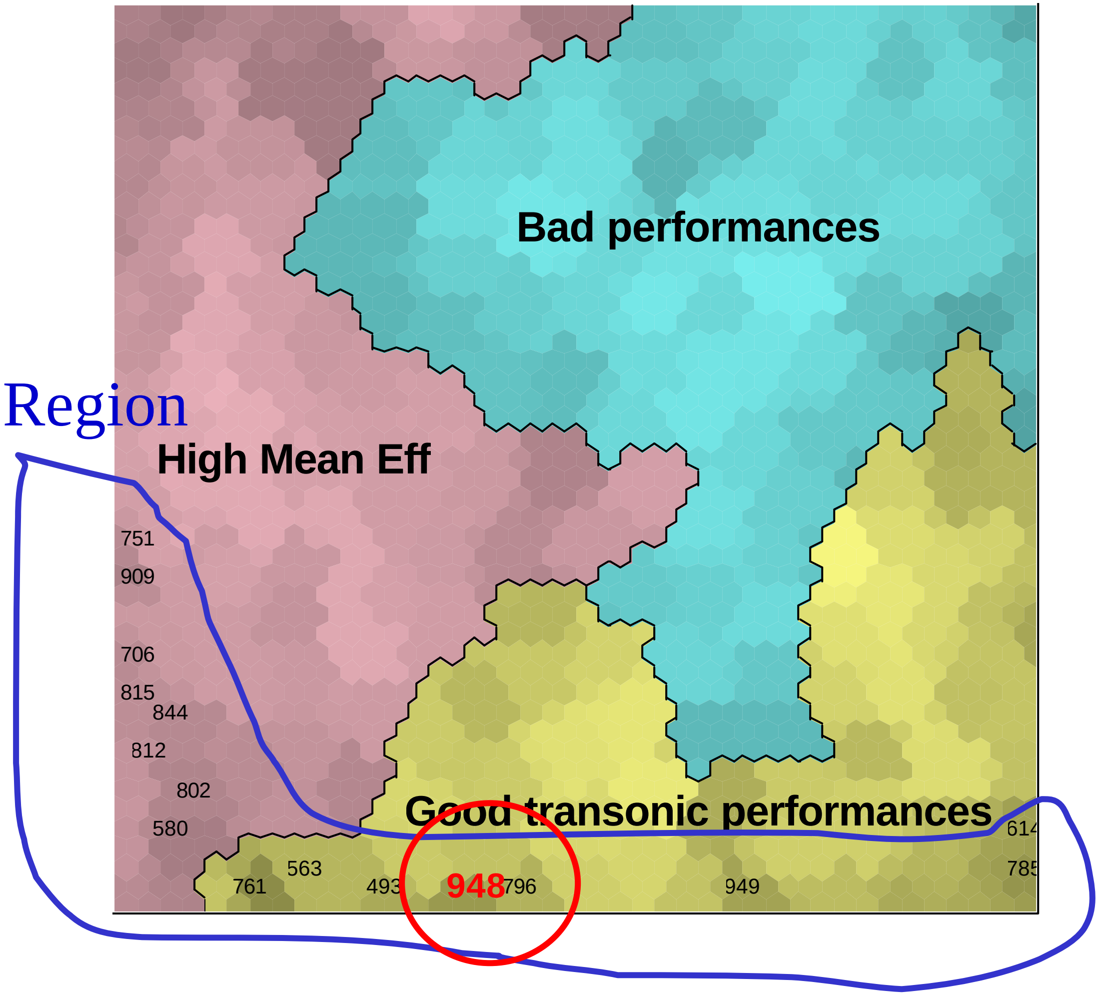
Objectives	Rae2822	Sol 948
E(Eff)	54,89	54,92
E(Cd)	1,173E-02	1,128E-02
Dev(Eff)	1,766	1,593
Dev(Cd)	1,583E-03	1,316E-03

Better results for stability

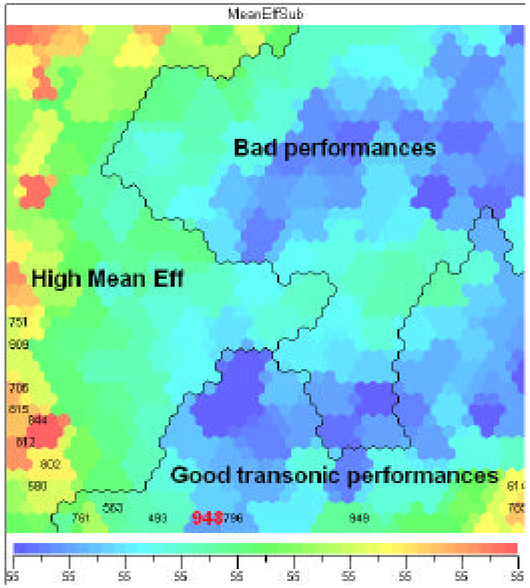
Visualization in Multi-D (SOM)

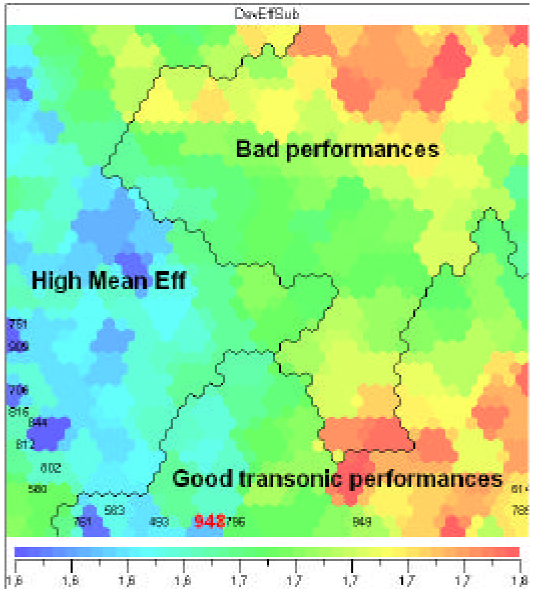
It is possible to rappresent all the feasible configuration with 3 clusters, with the Pareto Front and MCDM best configuration

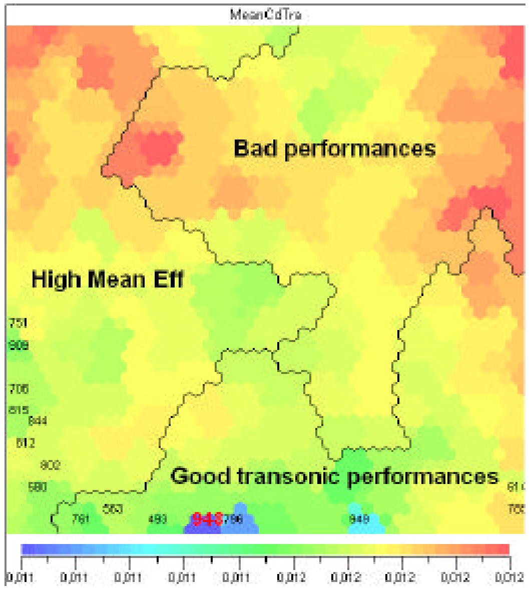
Pareto Front Region

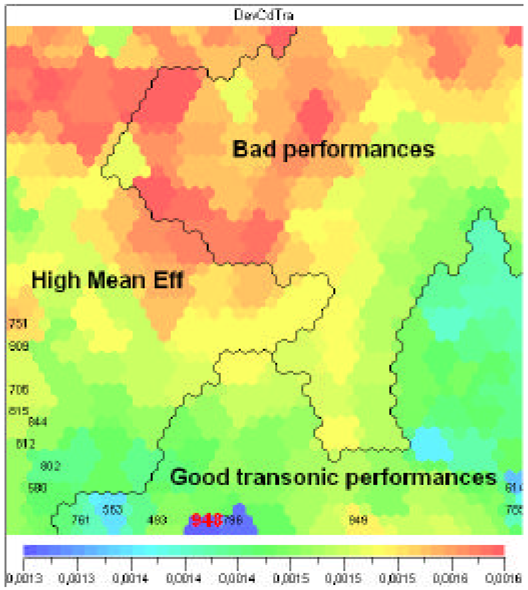


Visualization in Multi-D (SOM)









The names of the clusters are easy to fix

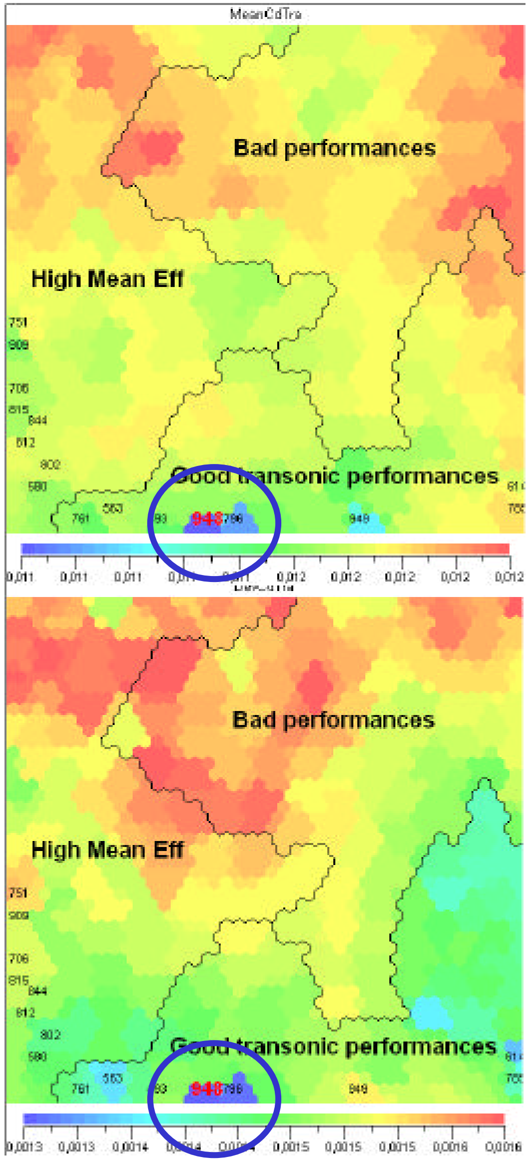
Inverse correlation between MeanEffSub and DevEffSub

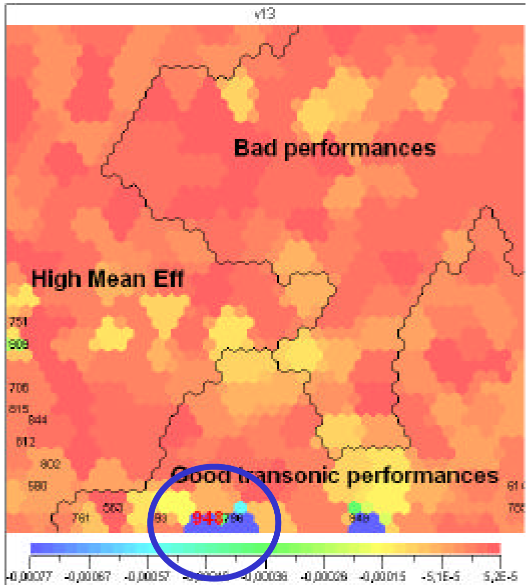
No visible correlation between EffSub and CdTra

MCDM finds the best transonic performances

Visualization in Multi-D (SOM)

Correlations between design variables and performances





Correlation between V13 and transonic performances

Conclusion

- **Robust Design is highly innovative since it considers the stability of one solution;**

- **When a Robust Design Optimization is needed, the best way is a Multi Objective Algorithm (Game Theory: Nash approach);**

- **The use of SOM is very interesting for the visualization in Multi-D space (Multi Objective, multi variables)**