2013/12/09 NAFEMS Japan Conference

Development of User-Subroutines for Constitutive law by the cooperation of Industrial users, Software vendors and Academic researchers

JANCAE - Japan Association of NonLinear CAE

Hideo Takizawa, Nippon Institute of Technology

Activities of NPO JANCAE

- JANCAE.org

JANCAE 特定非営利活動法人・非線形CAE協会

最新情報

2013/3/11
第23期非線形CAE勉強会のページを作成しました。
2013/1/15
ホームページをリニューアルいたしました。
2012/9/20

非線形計算力学分野における「世界標準」！！

2000年代前半までに体系化された非線形材料のための有限要素法を正しく学べる，従来の和書とは一線を画した計算力学分野における「世界標準」の一冊です。

「非線形有限要素法 弾塑性解析の理論と実践」
EA de Souza Neto　D Peric　DRJ Owen／原著　寺田賢二郎／監訳
森北出版株式会社

第23期非線形CAE勉強会のお知らせ

■基礎勉強会：2013年5月17日（金）
@東海大学高輪キャンパス 2号館 2201+2202教室
■1，2日目：2013年5月18・19日（土・日）
@東海大学高輪キャンパス 大講義室
■3，4日目：2013年6月1・2日（土・日）
@中央大学駿河台記念館

詳細はこちら

2012年度 第3回材料モデリング分科会のお知らせ

■日時：2013年3月14日（木）10時～17時
■会場：中央大学駿河台記念館　330教室

詳細はこちら

- President : Prof.Kenjiro Terada (Tohoku Univ)
- Holding CAE training courses 2tims/year as opportunities for companies, universities and software vendors to learn non-linear CAE
- Industrial companies usually use commercial CAE codes, and the deeper understanding is necessary for effective use of CAE

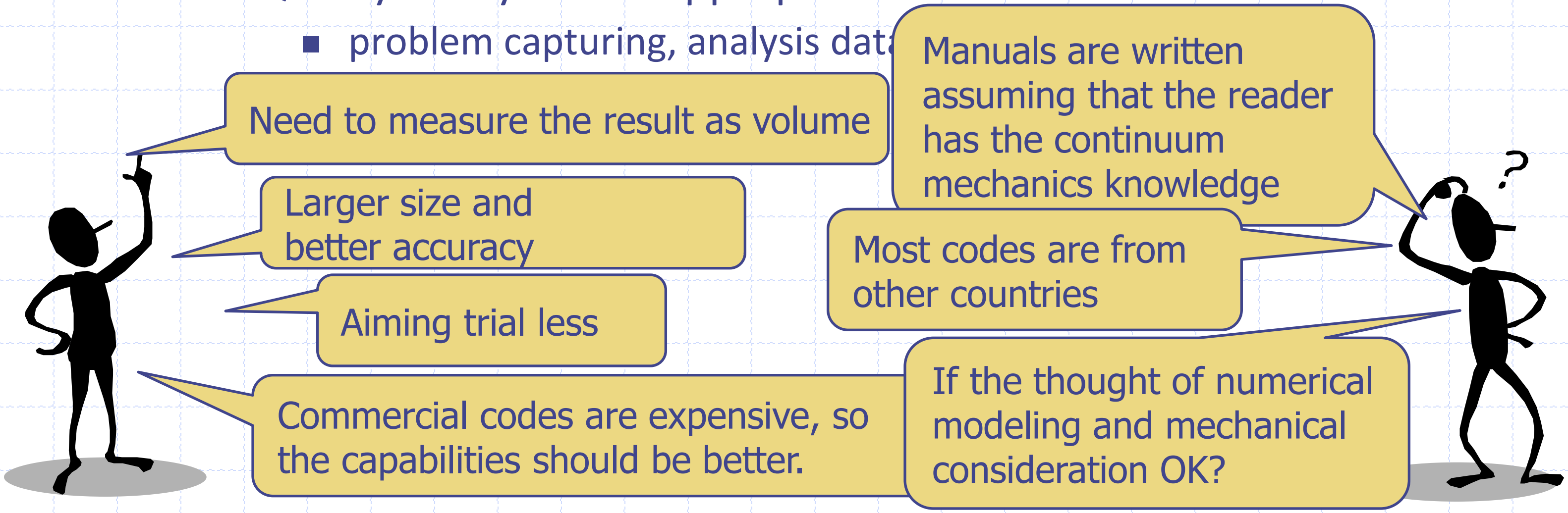
Activities of NPO JANCAE

- Nonlinear CAE training courses: 24 times for the past 12 years
  - The number of participants
    - 200 in Tokyo, 2 weekends in May and June
    - 120 in Nagoya, 2 weekends in November and December
  - The participants
    - Industrial engineers of 30’s and 40’s (They pay the fee by themselves, not by companies)
    - Vendors engineers and academic related

Why 200 engineers gather for 2 weekends!?

Activities of NPO JANCAE

- Position in companies：CAE as “volume (Quantity)”
  - If using CAE…(*)… maybe the cost for prototype test will be decreased.
    - If improving analysis accuracy, “trial-less” design and manufacturing will be possible.
    - There are things between lines(*)
- Personal engineer：CAE as “Quality”
  - As valued by working volume, they need to deal with calculations
    - Volume: the number of analysis case, analysis accuracy, the number of license, the rate of operation…
  - Free-floating anxiety about “Quality”
    - Quality: Is my model appropriate ?
      - problem capturing, analysis dat



Activities of NPO JANCAE

- Participants needs
  - Vendors seminars：Usually how to use the commercial software codes
    - Valid for beginners, but not involved deeply in the real problems.
    - Discussion is about specific software: How To Use/Push Button.
  - Universities/Academic societies：Basic theory and result of leading research
    - General basic theory： Already embedded in commercial codes
    - Leading research result：Can’t use with a little study

- How to use knowledge we have learned?
- 3 Non Linearity
  - Geometric NL
    - Theoretically completed. Difficult to be involved as users
  - NL on Boundary condition
    - Related to core of the codes. Difficult to be involved as users
  - Material NL
    - An enormous amount of material model, material test method Parameter identification
    - Both vendors and users don’t want to be involved?

Learn correctly

Model parameter and verification

Learn in proper order

Activities of NPO JANCAE

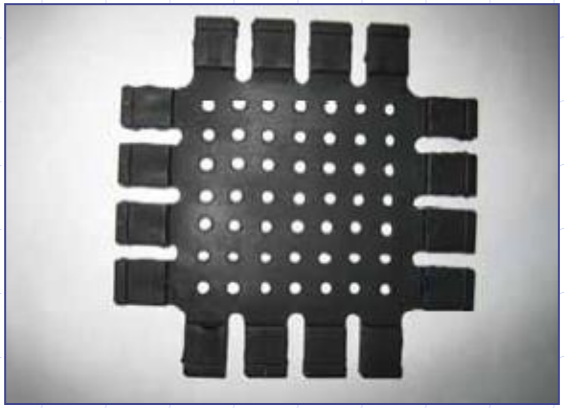
- Material modeling committee（Rubber committee）
  - Small group (30～60)
  - Specific topics for deeper understanding and effectiveness in practical aspect

When we think of comprehensive advancements in the accuracy of a simulation, we are aware that all capabilities of the material modeling, the boundary conditions as well as the definition of the geometric modeling have to be improved at the same level. The capabilities of geometric modeling for FEM simulations have drastically improved along with the growth of the 3D CAD market, the advancements in auto-meshing capabilities and the progresses made in hardware speeds and capacities, over the past 10 years. Yet material modeling capabilities have not progressed as significantly as the advances achieved in geometric modeling. Users need to be involved in the definition process of material modeling, which means that they have to choose the appropriate material model from huge amounts of available material models offered by each FEM code. As a next step, the parameters of the material properties have to be determined by performing material tests. These processes are still necessary, even now, at a time when many sophisticated commercial FEM codes are available.

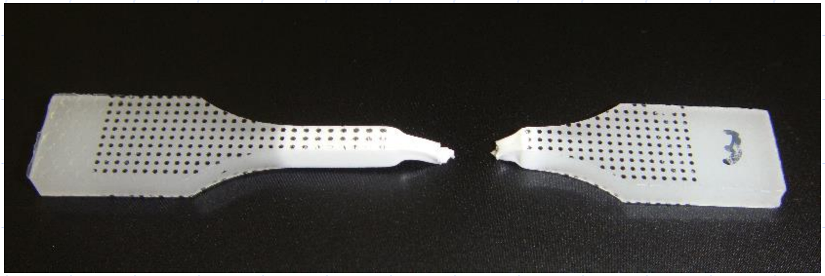
In this situation and independently from its CAE training course, which mainly consists of classroom lectures, JANCAE organizes “The Material Modeling Committee” as a practical approach to the study of nonlinear materials. The Committee was originally established in 2005 to study mainly hyperelasticity and viscoelasticity. Then, its research activities have diversified into all material nonlinearity including metal plasticity. In the frame of the Committee, members learn about typical nonlinear material modeling by studying the basic theory of the constitutive equations, material testing methods, and how to handle test data and parameter identification techniques.

Activities of NPO JANCAE

- Activities of material modeling committee
    - Collaboration and data sharing for nonlinear material test
        - Opened to the public
    - Training course for theory of constitutive law
        - Elastic･HyperElastic･ViscoElastic･Plastic･Creep…
        - Constitutive law developed in different fields （Metallurgy/Chemical/Process)



Rubber biaxial tensile test piece



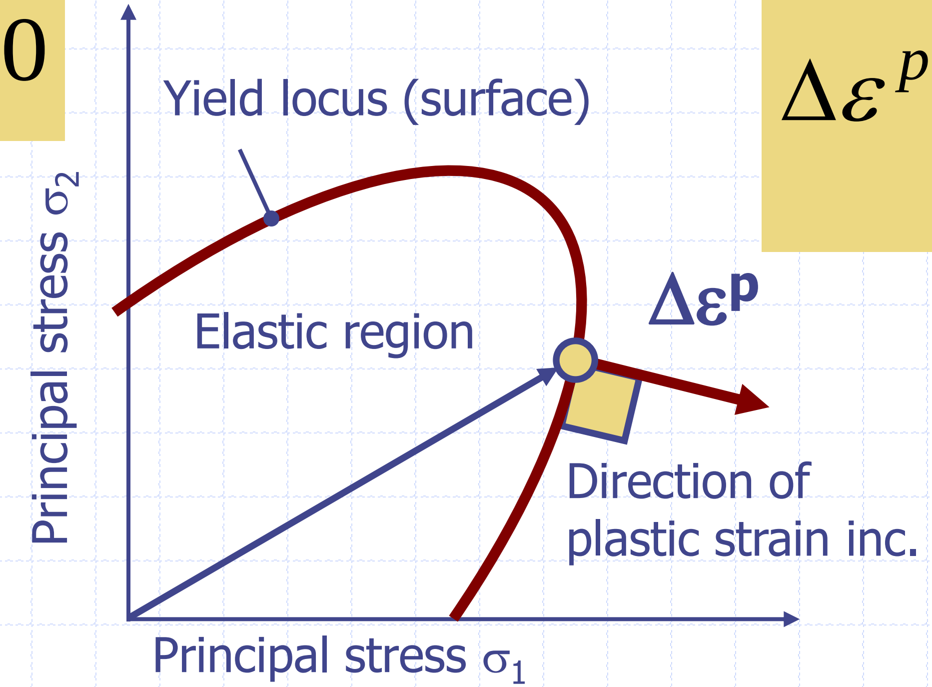
High speed uni-axial tensile test of polymer resin

- Activities to practically use what we learned practically

Basic for plastic theory

- Keyword to understand the theory of plasticity
  - Yield surface/Yield function
    - Assuming the function of stress components describing a surface in which the material is elastic, and on which plastic.
  - Normality rule ：Condition of stable plastic deformation
    - Direction of plastic strain increment is same as normal on the yield surface

$$\sigma_e(\sigma_{ij}) = 0$$



$$\Delta\varepsilon^p = \Delta p \frac{\partial \sigma_e}{\partial \sigma_{ij}}$$

Commercial CAE codes:

- **LS-DYNA(v972)**
  - TYPE 3,12,24 : vonMises
  - TYPE 33,36 : Barlat(1991), Barlat(1996)
  - TYPE 36 : Barlat(1989)
  - TYPE 37,39,103,122 : Hill(1948)
  - TYPE 133 : Barlat(2000) …..
- **MSC.Marc(2008)**
  - VON MISES : von Mises
  - HILL : Hill(1948)
  - BARLAT : Barlat(1991)
- **Abaqus(6.8), Adina, Ansys(11)**
  - von Mises
  - Hill(1948)

Many anisotropic yield functions are prepared in LS-DYNA which focus on sheet metal forming as explicit method. However, implicit codes only have classical anisotropic yield functions.

- User defined material model
  - Commercial codes provide user sub routine for material model
- Self-study has a limit (too difficult to learn alone !)
  - Backward Euler stress integration
  - Consistent tangent coefficient
  - Code dependent manner
- Both require lots of effort to users

Big gap between academic and industrial use

Basic theory of elasto-plastic constitutive law

Basic formula (Isotropic hardening)

- Voigt notation is used to fit to the user subroutine

$\sigma_e(\sigma)$ **: Yield Function**

$\sigma_Y(p)$ **: Hardening Curve**

Additive Decomposition $\{\Delta\varepsilon\} = \{\Delta\varepsilon^p\} + \{\Delta\varepsilon^e\}$

Hooke's law $\{\Delta\sigma\} = [D^e]\{\Delta\varepsilon^e\}$

Normality rule $\{\Delta\varepsilon^p\} = \Delta p \dfrac{\partial\sigma_e}{\partial\{\sigma\}} = \Delta p \dfrac{\partial\sigma_e}{\underline{\partial\{\sigma_{n+1}\}}}$ **Backward Euler**

Yield criterion $\sigma_e(\underline{\sigma_{n+1}}) - \sigma_Y(\underline{p_{n+1}}) = \sigma_e(\sigma_{n+1}) - \sigma_Y(p_n + \Delta p) = 0$

Backward Euler

Elastic predictor – Plastic Corrector

$$\{\Delta\sigma\} = [D^e]\{\{\Delta\varepsilon\} - \{\Delta\varepsilon^p\}\} = [D^e]\{\Delta\varepsilon\} - [D^e]\Delta p \frac{\partial\sigma_e}{\partial\{\sigma_{n+1}\}}$$

$$\{\sigma_{n+1}\} = \{\sigma_n\} + \{\Delta\sigma\} = \underline{\{\sigma_n\} + [D^e]\{\Delta\varepsilon\}} - [D^e]\Delta p \frac{\partial\sigma_e}{\partial\{\sigma_{n+1}\}}$$

Trial stress $\{\sigma^{Try}\}$

Elasto-plastic model: Stress integration

- Plandtl-Reuß (Analytical form)
  - L.Plandtl (1924)
  - E.Reuss (1930)

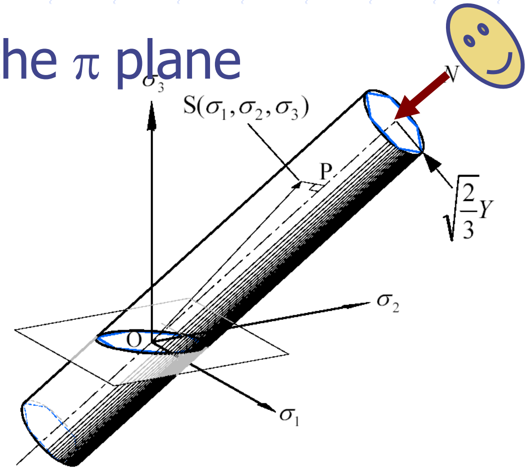
- Forward Euler (Tangential Predictor/r-min method)
  - P.V.Marcal, I.P.King (1967)
  - Y.Yamada(1967)
  - Y.Yamada, N.Yoshimura, T.Sakurai (1968)

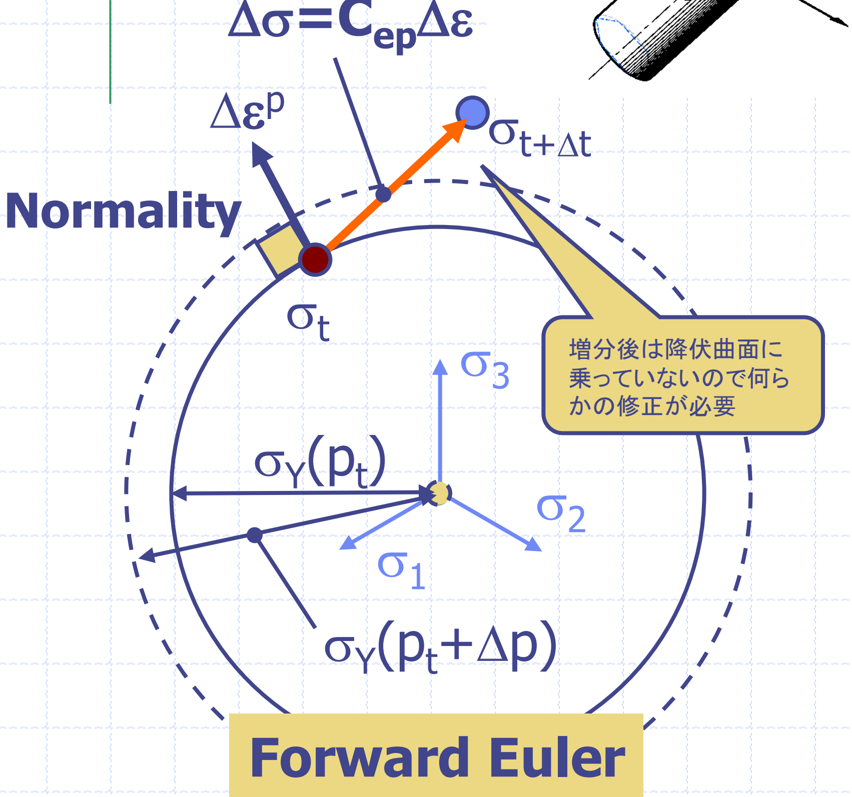
- Mean Normal Method
  - J.C.Nagtegaal, J.E.De Joung (1980,1981)
  - I.Pillinger, P.Hartley, C.E.N.Sturgess, G.W.Rowe (1986)

- Backward Euler (Elastic predictor-Return Mapping)
  - R.D.Krieg, D.B.Krieg (1977)
  - J.C.Nagtegaal (1982)
  - J.C.Simo, M.Ortiz (1985) : (Consistent Tangent)
- Mutistage Return Mapping
    - M.Ortiz, J.C.Simo (1989)

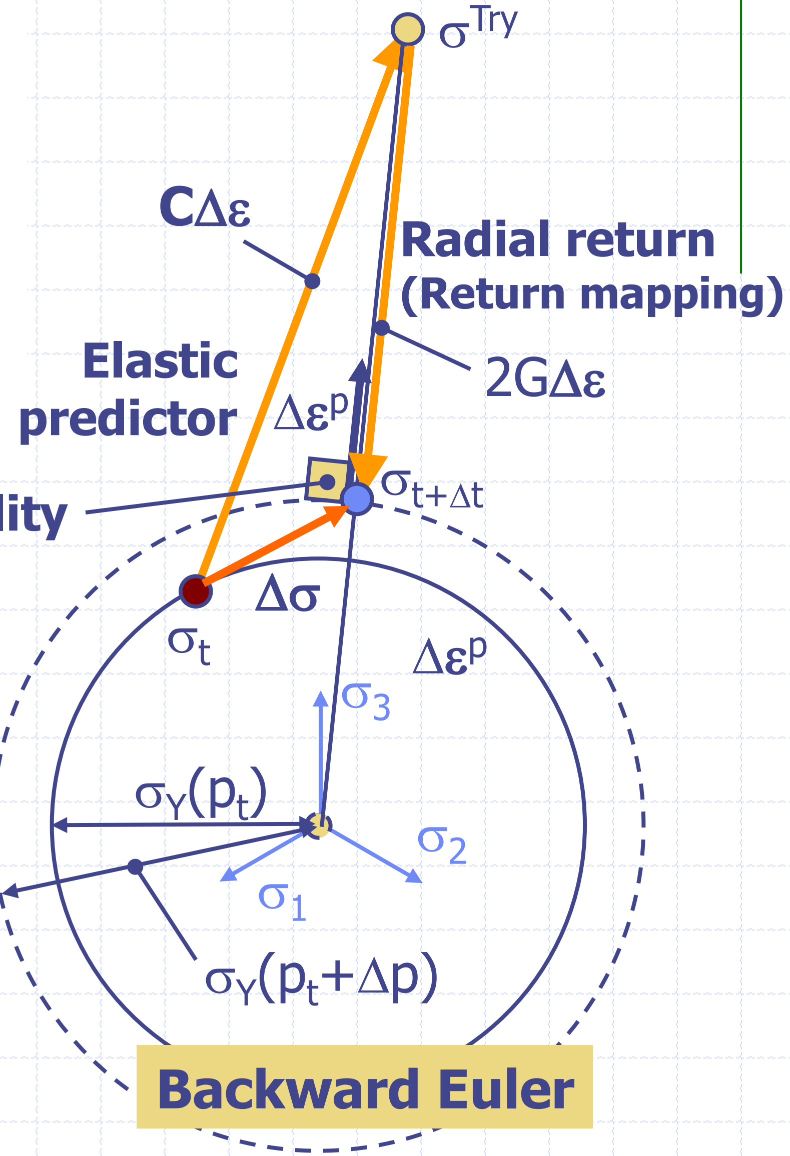
Forward/Backward Euler

- Description on the π plane





増分後は降伏曲面に乗っていないので何らかの修正が必要



Basic theory

- Update of stress

$$[E]\equiv\left[[I]+\Delta p_{(i)}[D^e]\boxed{\frac{\partial^2\sigma_e}{\partial\{\sigma_{(i)}\}\partial\{\sigma_{(i)}\}^{\mathrm{T}}}}\right]$$

NR loop

δΔp (Perturbation amount of Δp)

$$\delta\Delta p=\frac{g_1(\Delta p_{(i)})-\boxed{\frac{\partial\sigma_e}{\partial\{\sigma_{(i)}\}^{\mathrm{T}}}}[E]^{-1}\{g_2(\Delta p_{(i)})\}}{\boxed{\frac{\partial\sigma_e}{\partial\{\sigma_{(i)}\}^{\mathrm{T}}}}[E]^{-1}[D^e]\boxed{\frac{\partial\sigma_e}{\partial\{\sigma_{(i)}\}}}+\boxed{\frac{d\sigma_Y}{dp_{(i)}}}}$$

δσ (Perturbation amount of σ)

$$\{\delta\sigma\}=-[E]^{-1}\left\{\{g_2(\Delta p_{(i)})\}-[D^e]\delta\Delta p\boxed{\frac{\partial\sigma_e}{\partial\{\sigma_{(i)}\}}}\right\}$$

Update

$$\Delta p_{(i+1)}=\Delta p_{(i)}+\delta\Delta p$$

$$\{\sigma_{(i+1)}\}=\{\sigma_{(i)}\}+\{\delta\sigma\}$$

Check residuals

$$g_1(\Delta p_{(i+1)})=\boxed{\sigma_e(\sigma_{(i+1)})}-\boxed{\sigma_Y(p_n+\Delta p_{(i+1)})}$$

$$\{g_2(\Delta p_{(i+1)})\}=\{\sigma_{(i+1)}\}-\{\sigma^{Try}\}+\Delta p_{(i+1)}[D^e]\boxed{\frac{\partial\sigma_e}{\partial\{\sigma_{(i+1)}\}}}$$

σ_e and its differential

σ_y and its differential

Convergence condition

g_1,norm$\{g_2\}$<Tol

Basic theory

- Consistent tangent modulus

$$\{\delta\sigma\}=\left[[e]^{-1}-\frac{[e]^{-1}\boxed{\dfrac{\partial\sigma_e}{\partial\{\sigma_{n+1}\}}}\otimes[e]^{-1}\boxed{\dfrac{\partial\sigma_e}{\partial\{\sigma_{n+1}\}}}}{\boxed{\dfrac{\partial\sigma_e}{\partial\{\sigma_{n+1}\}}}[e]^{-1}\boxed{\dfrac{\partial\sigma_e}{\partial\{\sigma_{n+1}\}}}+\boxed{\dfrac{d\sigma_Y}{dp_{n+1}}}}\right]\{\delta\varepsilon\}$$

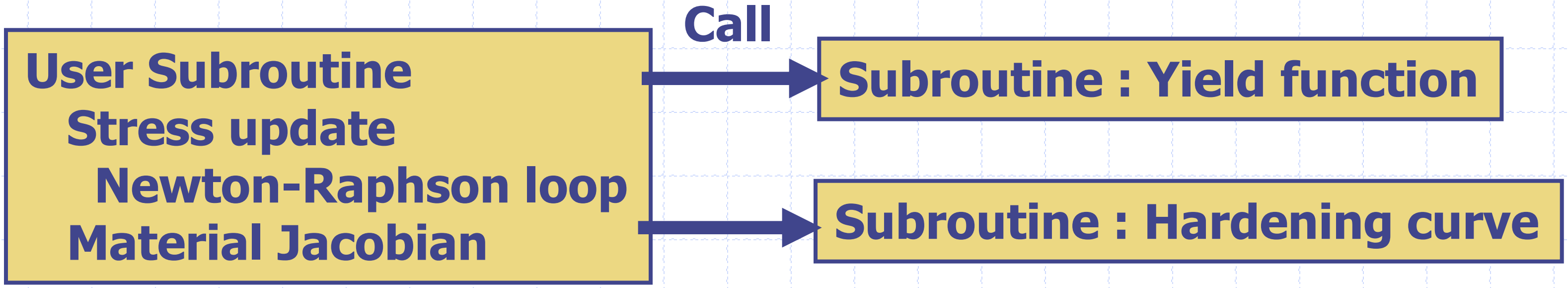
$$[e]=\left[[D^e]^{-1}+\Delta p\boxed{\dfrac{\partial^2\sigma_e}{\partial\{\sigma_{n+1}\}\partial\{\sigma_{n+1}\}^{\mathrm{T}}}}\right]$$

$\sigma_e$and its diffrential

$\sigma_y$and its differential

Generalization of plastic constitutive law

- Given yield function (equivalent stress) and strain hardening law
  - For the given stress{σ}
    - Equivalent stress σ_e
    - First order differential by stress of equivalent stress $\partial\sigma_e / \partial\{\sigma\}$
    - Second order differential by stress of equivalent stress $\partial^2\sigma_e / \partial\{\sigma\}\partial\{\sigma\}^{\mathrm{T}}$
  - And the given equivalent plastic strain p
    - Deformation resistance σ_Y
    - First order differential by p of deformation resistance $d\sigma_Y / dp$
  - Should to be returned.



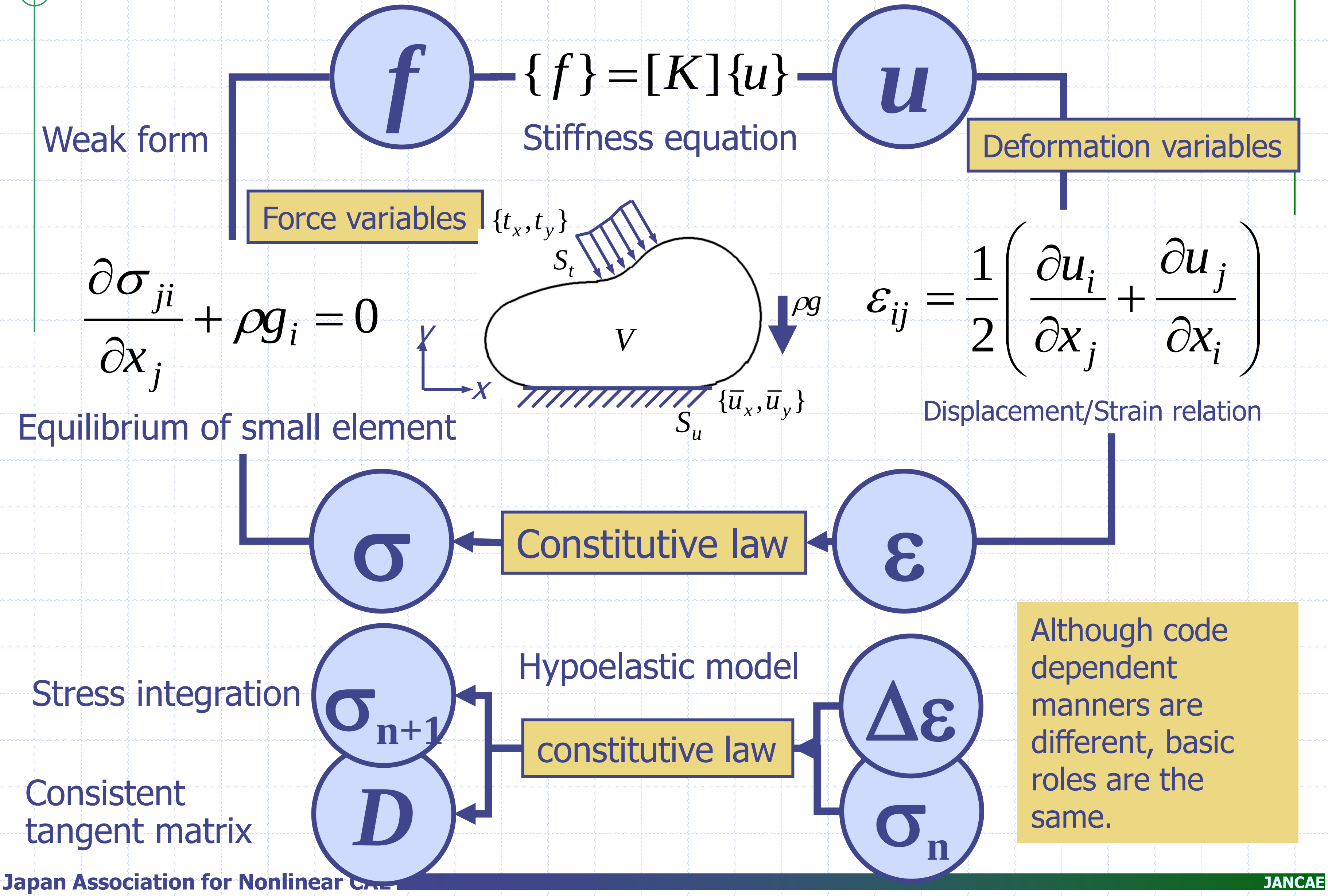
User Subroutine
Stress update
Newton-Raphson loop
Material Jacobian

Call

Subroutine : Yield function

Subroutine : Hardening curve

Role of constitutive law sub routine



$$\{f\} = [K]\{u\}$$

Stiffness equation

Weak form

Force variables

Deformation variables

$$\frac{\partial \sigma_{ji}}{\partial x_j} + \rho g_i = 0$$

Equilibrium of small element

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Displacement/Strain relation

Constitutive law

Hypoelastic model

Stress integration

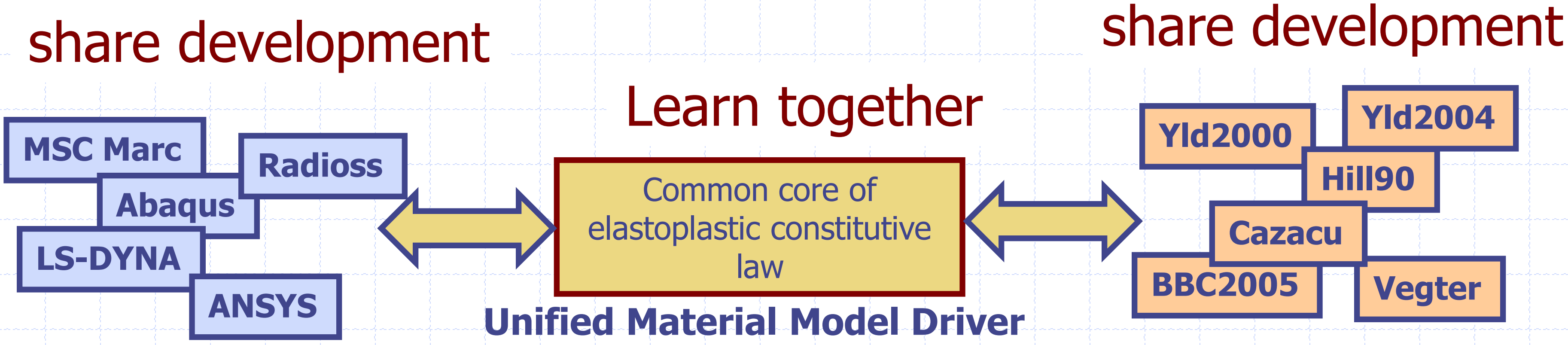
constitutive law

Consistent tangent matrix

Although code dependent manners are different, basic roles are the same.

Externality of 2 kinds of diversity

- Diversity of commercial codes
  - User subroutine of commercial codes has different specifications.
    - User subroutine is discussed in the users meetings and seminars held by vendors.
  - However, role of constitutive law is common in continuum mechanics.
    - MSC Marc and Abaqus have the same manner to store array
- Diversity of yield functions
  - There are many proposals of yield functions from many researchers.
  - However, numerically, $\sigma_e(\sigma)$ and its differential are necessary for expression of yield function in codes.
    - Can be extended if being module by subroutine
  - Same for hardening law and hardening curve



share development

MSC Marc
Radioss
Abaqus
LS-DYNA
ANSYS

Learn together

Common core of elastoplastic constitutive law

Unified Material Model Driver

share development

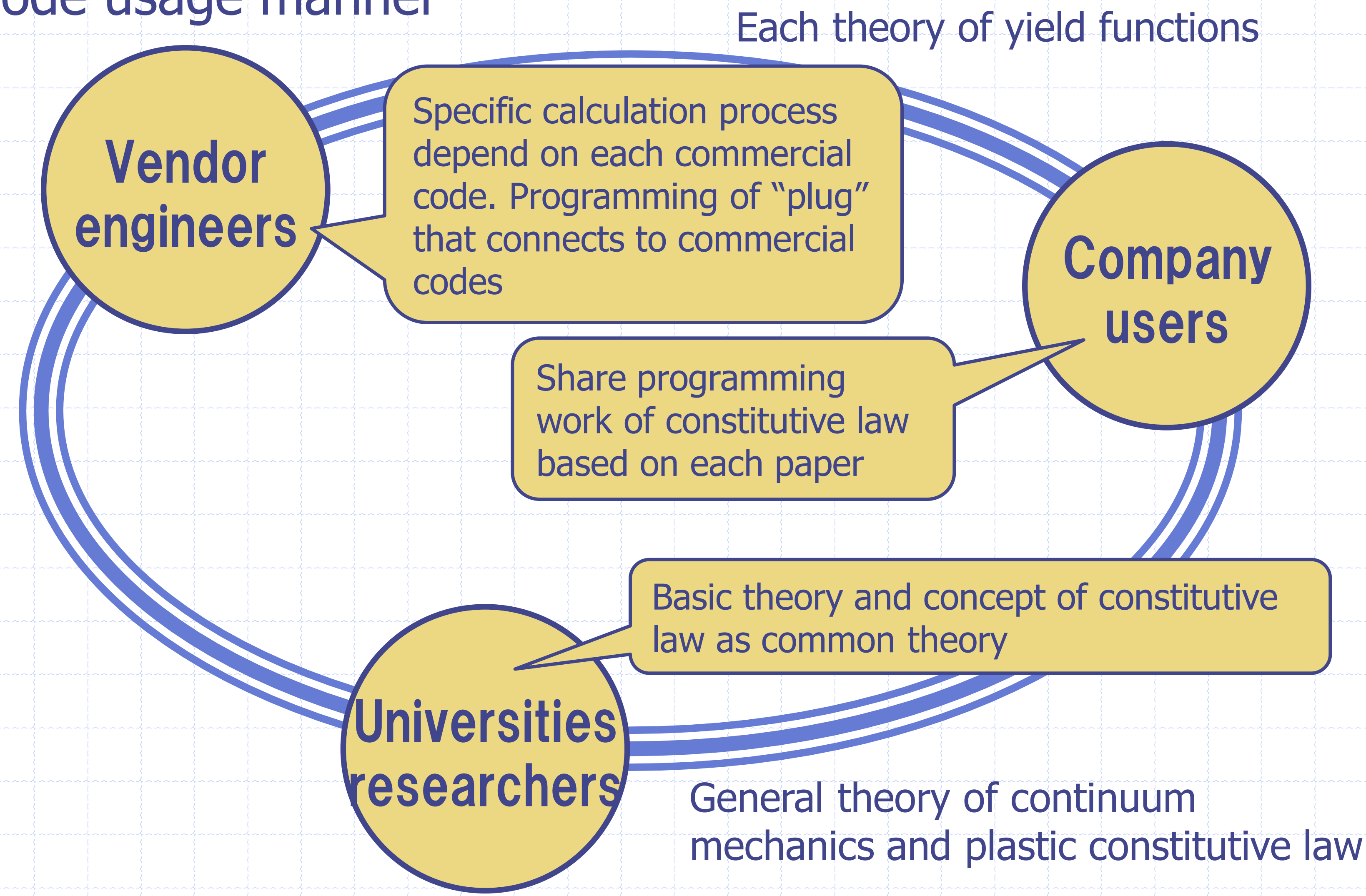
Yld2000
Yld2004
Hill90
Cazacu
BBC2005
Vegter

The aim of the working group

- Undestand
    - Industrial companies tend to have tools that we can use easily for the short term benefit. But it is difficult to get a good result depend on understanding by using black box tool.
        - Constitutive law's role is independent from codes
        - So we can learn together
- Create
    - Gaining peers understanding by setting clear target "user subroutine and peripheral programs"
        - Creating subroutine group which can be embedded in commercial codes
        - By doing so, creating basic technology for higher analysis
- Deliver
    - Deliver the result so that successive engineers can follow the same process and much more engineers can aspire higher goal
        - JANCAE is a NPO and emphasize public benefit

The working group structure

Code usage manner



Each theory of yield functions

Vendor engineers

Specific calculation process depend on each commercial code. Programming of “plug” that connects to commercial codes

Company users

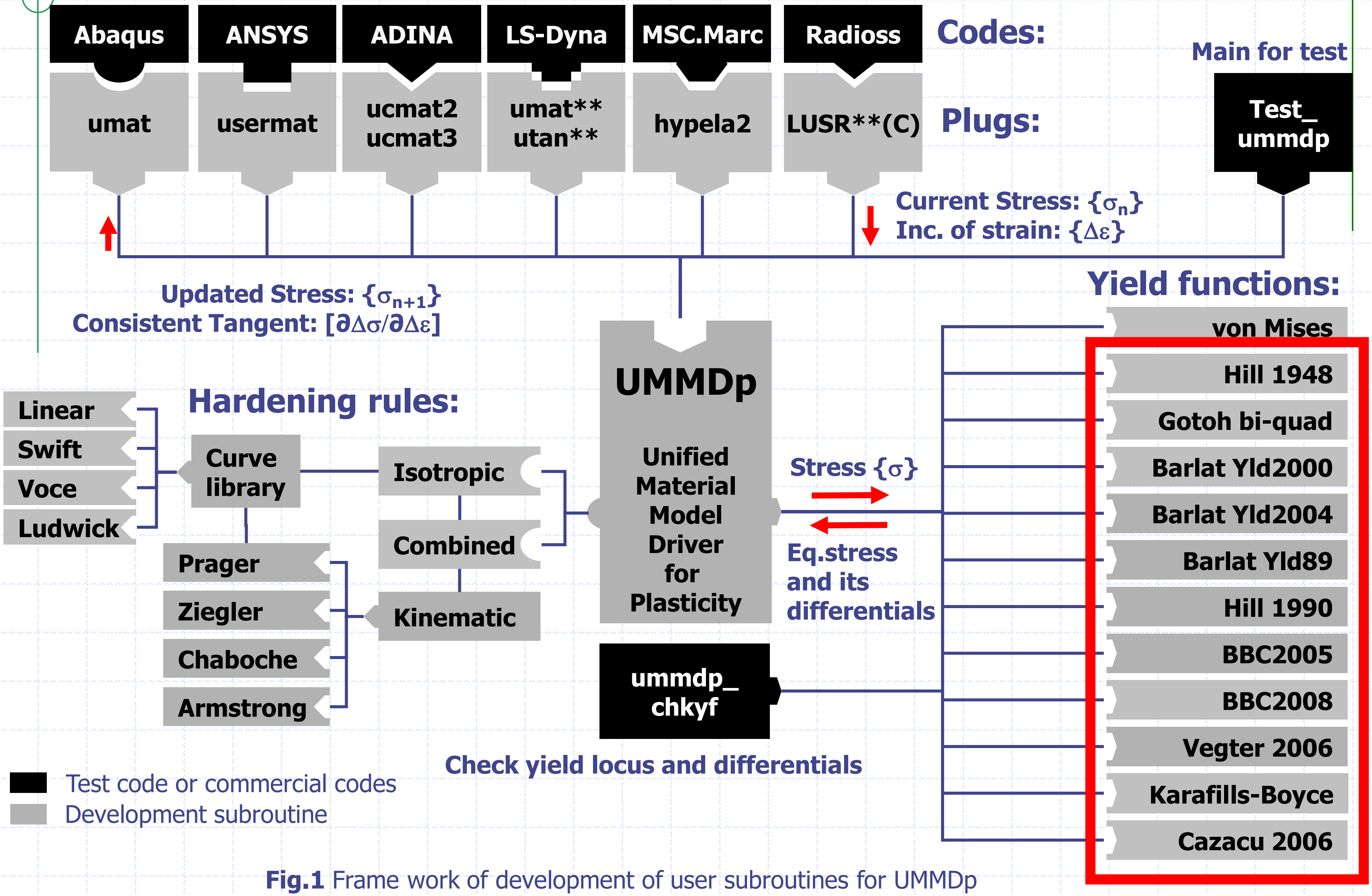
Share programming work of constitutive law based on each paper

Universities researchers

Basic theory and concept of constitutive law as common theory

General theory of continuum mechanics and plastic constitutive law

Flame work of UMMDp



Codes: Abaqus, ANSYS, ADINA, LS-Dyna, MSC.Marc, Radioss

Plugs: umat, usermat, ucmat2 ucmat3, umat** utan**, hypela2, LUSR**(C)

Main for test

Test_ ummdp

Current Stress: $\{\sigma_n\}$
Inc. of strain: $\{\Delta\varepsilon\}$

Updated Stress: $\{\sigma_{n+1}\}$
Consistent Tangent: $[\partial\Delta\sigma/\partial\Delta\varepsilon]$

UMMDp

Unified Material Model Driver for Plasticity

Hardening rules:

- Curve library: Linear, Swift, Voce, Ludwick
- Isotropic
- Combined
- Kinematic: Prager, Ziegler, Chaboche, Armstrong

Stress $\{\sigma\}$

Eq.stress and its differentials

Yield functions:

- von Mises
- Hill 1948
- Gotoh bi-quad
- Barlat Yld2000
- Barlat Yld2004
- Barlat Yld89
- Hill 1990
- BBC2005
- BBC2008
- Vegter 2006
- Karafills-Boyce
- Cazacu 2006

ummdp_ chkyf

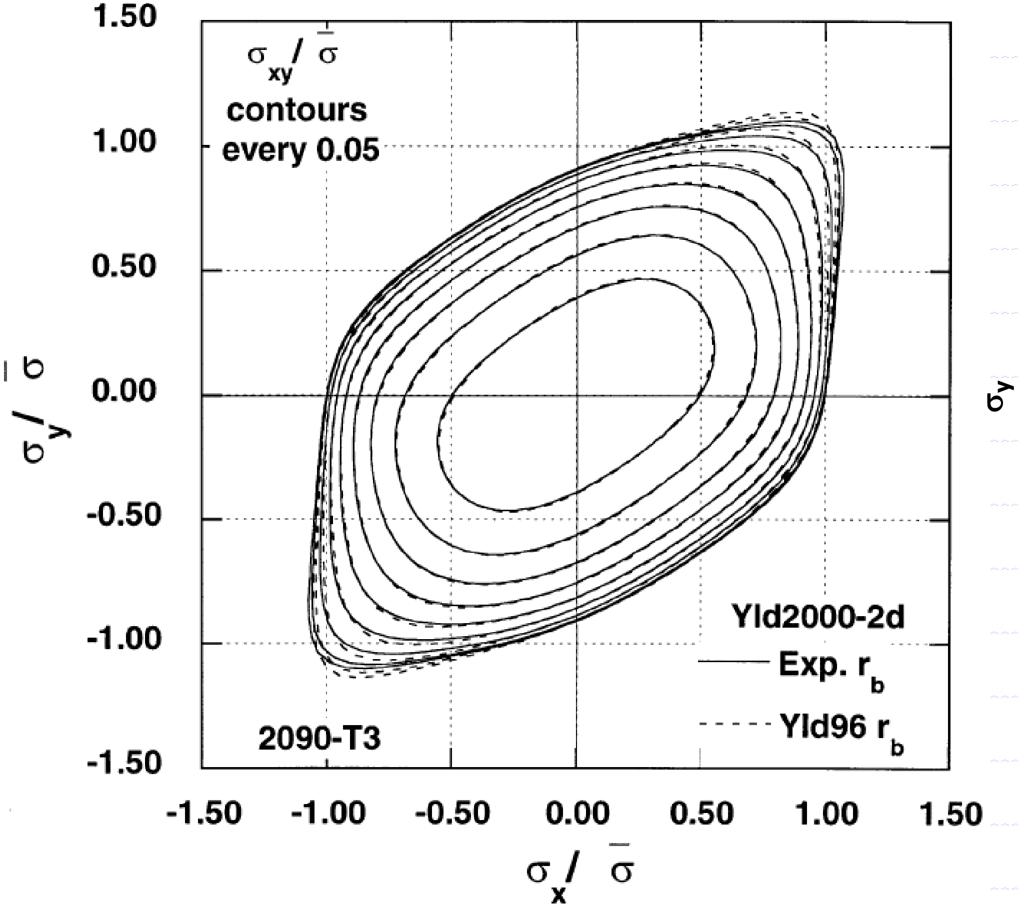
Check yield locus and differentials

■ Test code or commercial codes
▨ Development subroutine

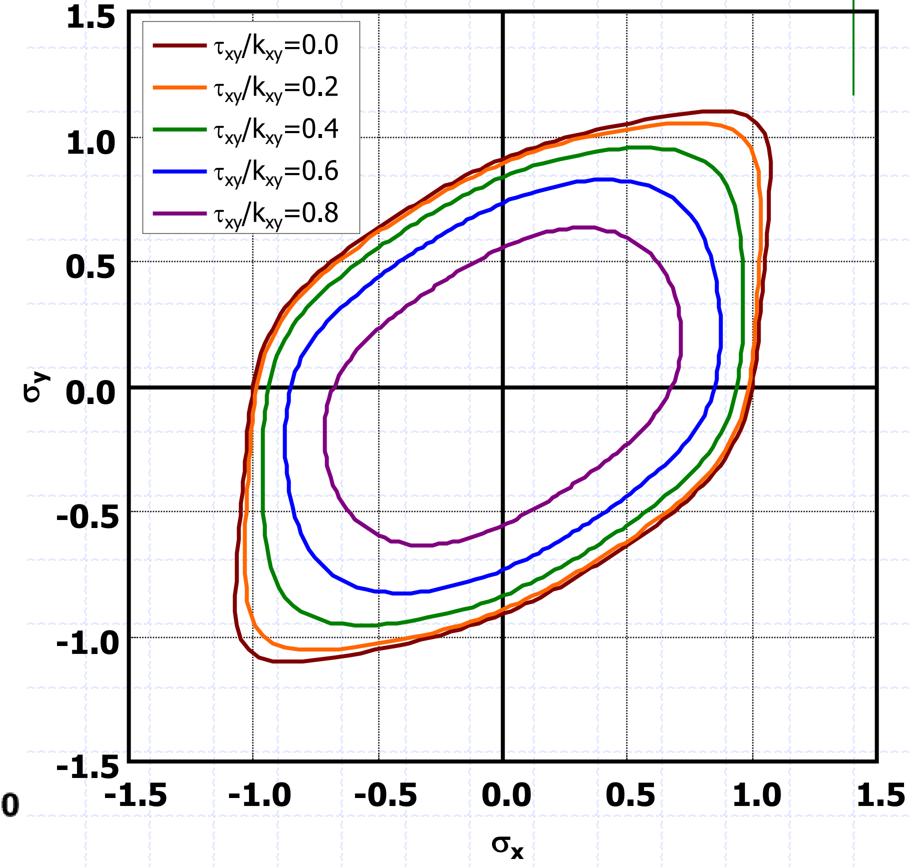
Fig.1 Frame work of development of user subroutines for UMMDp

Yield function : Yld2000

Barlat yld2k paper



Chkyf output xy.txt graph



Users members were in charge

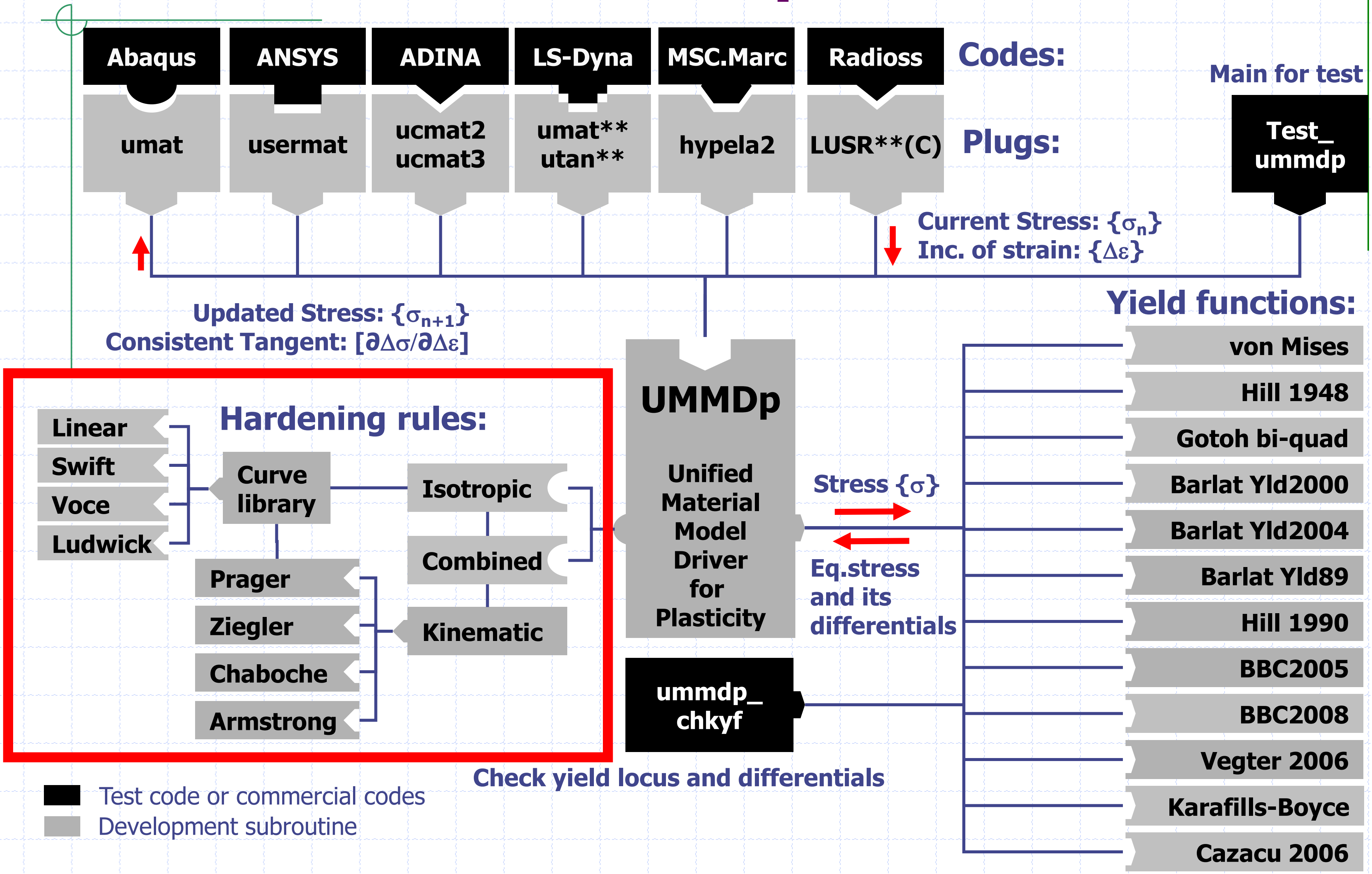
“Yield function subroutines” by Users

- Program by industrial users
  - Differ from vendor engineers, their understanding level and motivation are different
    - “I’m a process design engineer of metal forming.”
    - “I’m not involved in anisotropic, but it’s an good opportunity to study it.”
    - “I used Fortran for the first time in this project.”
    - “It is hard to read English papers.”

- Variety of environment and positions
  - Difficult to obtain understanding from companies
    - As a result, it was a winter vacation homework as self-development
    - Positive to open to public

- Thank you to all users who participates in the project

Flame work of UMMDp



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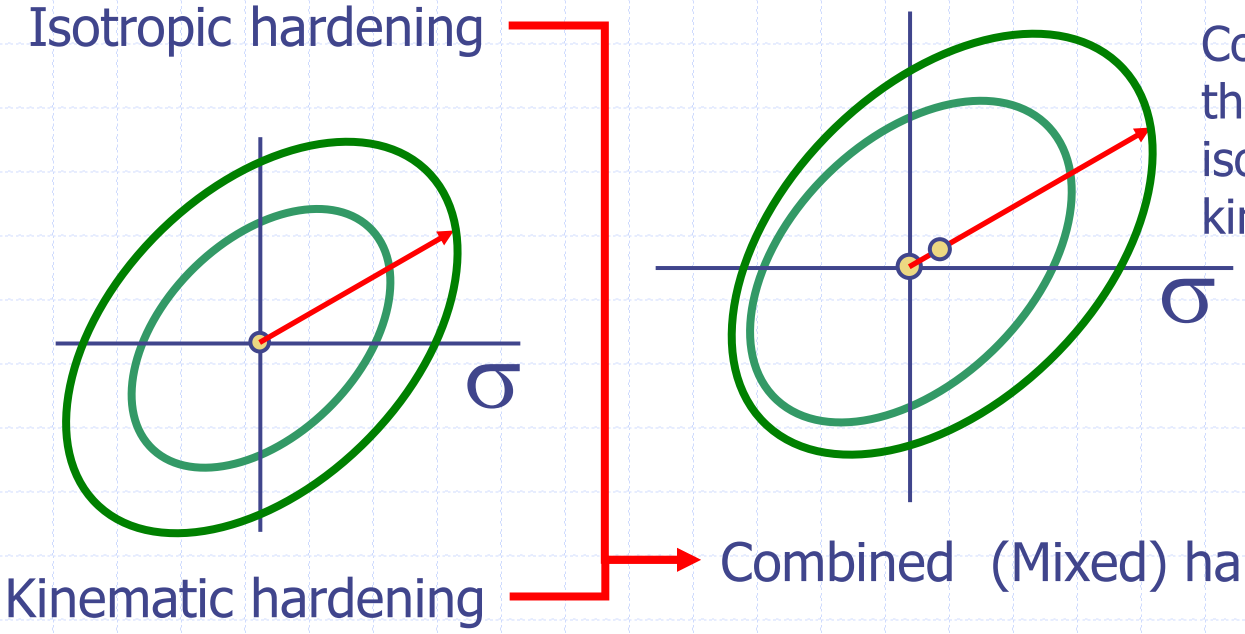
ummdp_ chkyf

Check yield locus and differentials

■ Test code or commercial codes
▨ Development subroutine

Fig.1 Frame work of development of user subroutines for UMMDp

Hardening law：Combined hardening model

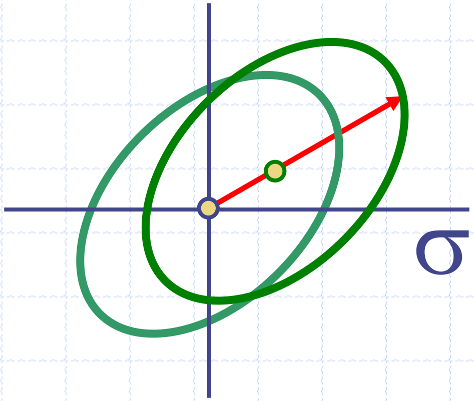


Isotropic hardening

Kinematic hardening

Combined (Mixed) hardening

Combined hardening is the superset of isotropic hardening and kinematic hardening.



Isotropic hardening and kinematic hardening are not exclusive options
Only one of those can not explain natural phenomenon perfectly. In reality, it’s correct to consider that these 2 hardening laws are being together.

例:「初期降伏応力の二倍の応力まで引張って除荷（応力をゼロに）した際に，逆降伏が生じなかったので，この金属は移動硬化ではなく，等方硬化である．」という話は一見正しそうに聞こえる．しかし．「完全な移動硬化」でないことを示してはいるが，「完全な等方硬化」であることの証明にはなっていない．

Basic theory of combined hardening

- Combined hardening is superset
  - Isotropic hardening and kinematic hardening are not exclusive options
  - Develop the combined hardening
- Basic equation
  - For user subroutine, Voigt notation is used
  - Use backward Euler is used same as isotropic hardening
  - Explain back stress as {X} and use evolution equation of back stress.

Additive Decomposition

$$\{\Delta\varepsilon\}=\{\Delta\varepsilon^{p}\}+\{\Delta\varepsilon^{e}\}$$

Hooke's law

$$\{\Delta\sigma\}=[D^{e}]\{\Delta\varepsilon^{e}\}$$

Normality rule

$$\{\Delta\varepsilon^{p}\}=\xcancel{\Delta p\frac{\partial\sigma_e}{\partial\{\sigma_{n+1}\}}}=\Delta p\frac{\partial\sigma_e}{\partial\{\sigma_{n+1}-X_{n+1}\}}$$

Yield criterion

$$\xcancel{\sigma_e(\sigma_{n+1})-\sigma_Y(p_{n+1})=0}\quad\sigma_e(\sigma_{n+1}-X_{n+1})-\sigma_Y(p_{n+1})=0$$

Eq. of evolution of X

$$\{\Delta X\}=\Delta p\{V(\sigma_{n+1},X_{n+1},p_{n+1})\}$$

Kinematic hardening model

Classic back stress model

Eq. of evolution of Back stress
General Form

$$\{\Delta X\} = \Delta p\{V(\sigma, X, p)\}$$

- W.Prager

$$\{\Delta X\} = c(p)\{\Delta\varepsilon^p\}$$

$$\Rightarrow \{V\} = c(p)\frac{\partial\sigma_e}{\partial\{\eta\}} = c(p)\{m\}$$

- H.Ziegler

$$\{\Delta X\} = c(p)\Delta p\{\{\sigma'\} - \{X\}\}$$

$$\Rightarrow \{V\} = c(p)\{\{\sigma'\} - \{X\}\}$$

- P.J.Armstrong-C.O.Frederick

$$\{\Delta X\} = c(p)\{\Delta\varepsilon^p\} - \Delta p\gamma(p)\{X\}$$

$$\Rightarrow \{V\} = c(p)\{m\} - \gamma(p)\{X\}$$

Increment of equivalent plastic strain (or plastic multiplier) can be calculated from increment of plastic strain, when using normality rule

$$\{\Delta\varepsilon^p\} = \Delta p\frac{\partial\sigma_e}{\partial\{\sigma - X\}}$$

Δp is included in the equation in some way, considering that back stress evolves in association with progress of plastic deformation

Basic theory of combined hardening

$$\{m_{n+1}\} = \left.\frac{\partial \sigma_e}{\partial\{\sigma - X\}}\right|_{n+1}$$

- Stress integration

Variables : $\sigma_{n+1}, X_{n+1}, \Delta p$

Constant : $\Delta\varepsilon, p_n, \sigma_n, X_n$

$$\{\sigma^{Try}\} \equiv \{\sigma_n\} + [D^e]\{\Delta\varepsilon\}$$

Simplification

Yield criterion

$$\sigma_e(\sigma_{n+1} - X_{n+1}) - \sigma_Y(p_{n+1}) = \sigma_e(\sigma_{n+1} - X_{n+1}) - \sigma_Y(p_n + \Delta p) = 0$$

Updated stress

Elastro-plastic additive decomposition, Hooke law

$$\{\sigma_{n+1}\} = \{\sigma_n\} + \{\Delta\sigma\} = \{\sigma_n\} + [D^e]\{\Delta\varepsilon\} - [D^e]\Delta p \left.\frac{\partial \sigma_e}{\partial\{\sigma - X\}}\right|_{n+1} \equiv \{\sigma^{Try}\} - \Delta p[D^e]\{m_{n+1}\}$$

Back stress

$$\{X_{n+1}\} = \{X_n\} + \{\Delta X\}$$

$$\boxed{\{\Delta X\} = \Delta p\{v\} = \Delta p\{V(\sigma_{n+1}, X_{n+1}, p + \Delta p)\}}$$

Back stress evolution equation

All those equations must be satisfied. Define following “g” as the error functions

$$g_1 = \sigma_e(\sigma_{n+1} - X_{n+1}) - \sigma_Y(p_n + \Delta p) \rightarrow 0$$

Scalar equation

$$\{g_2\} = \{\sigma_{n+1}\} - \{\sigma^{Try}\} + \Delta p[D^e]\{m_{n+1}\} \rightarrow \{0\}$$

Vector equation

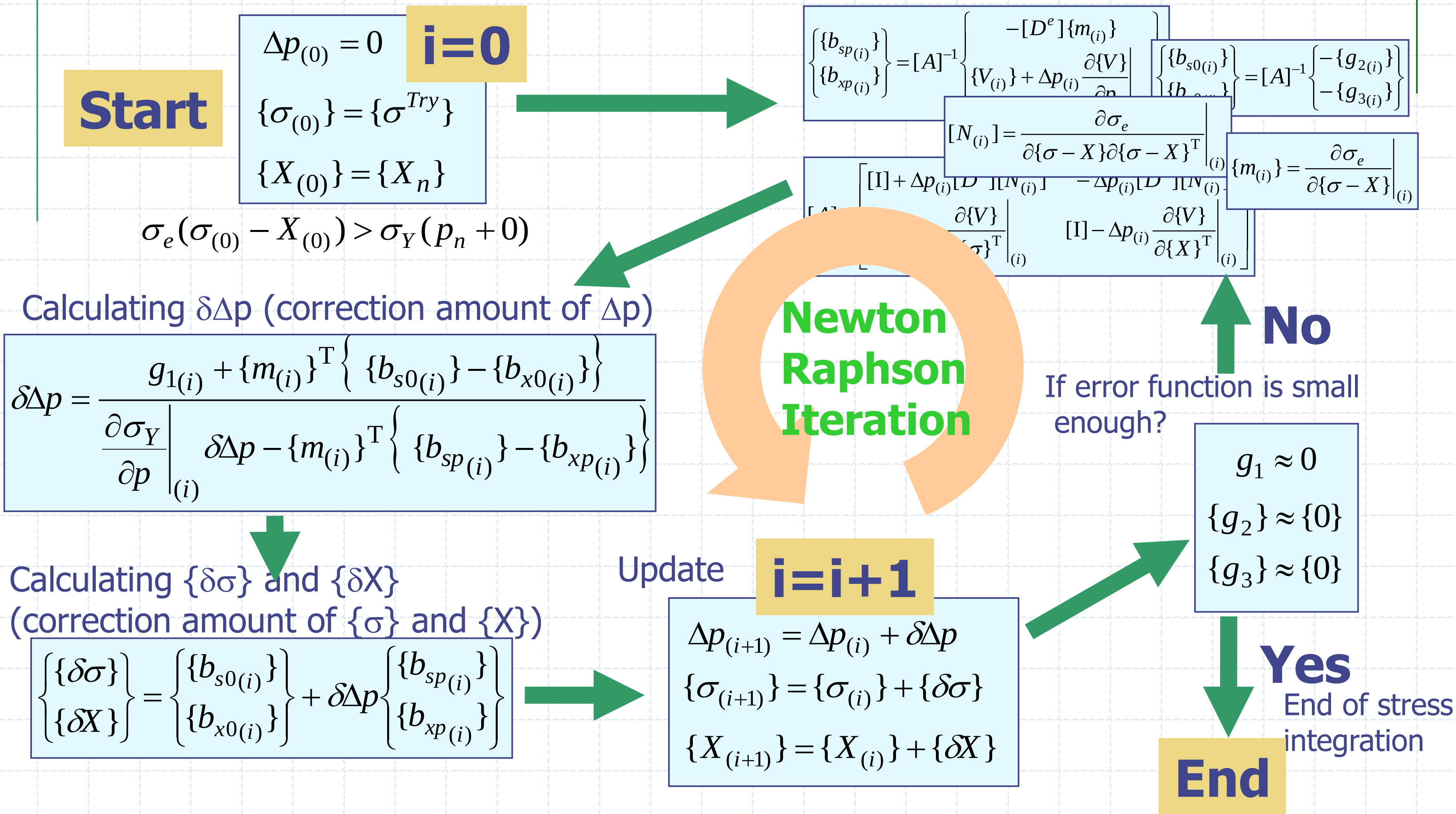
$$\boxed{\{g_3\} = \{X_{n+1}\} - \{X_n\} - \Delta p\{V(\sigma_{n+1}, X_{n+1}, p + \Delta p)\} \rightarrow \{0\}}$$

Vector equation

2n+1 simultaneous non-linear equation

Basic theory of combined hardening

◆ Summary of stress integration



Assumption of the default value (not yielding)

Start

i=0

$$\Delta p_{(0)} = 0$$

$$\{\sigma_{(0)}\} = \{\sigma^{Try}\}$$

$$\{X_{(0)}\} = \{X_n\}$$

$$\sigma_e(\sigma_{(0)} - X_{(0)}) > \sigma_Y(p_n + 0)$$

Calculation of intermediate variables

$$\begin{Bmatrix} \{b_{sp(i)}\} \\ \{b_{xp(i)}\} \end{Bmatrix} = [A]^{-1} \begin{Bmatrix} -[D^e]\{m_{(i)}\} \\ \{V_{(i)}\} + \Delta p_{(i)} \dfrac{\partial\{V\}}{\partial p} \end{Bmatrix}$$

$$\begin{Bmatrix} \{b_{s0(i)}\} \\ \{b \quad\} \end{Bmatrix} = [A]^{-1} \begin{Bmatrix} -\{g_{2(i)}\} \\ -\{g_{3(i)}\} \end{Bmatrix}$$

$$[N_{(i)}] = \left.\frac{\partial \sigma_e}{\partial\{\sigma - X\}\partial\{\sigma - X\}^{\mathrm{T}}}\right|_{(i)}$$

$$\{m_{(i)}\} = \left.\frac{\partial \sigma_e}{\partial\{\sigma - X\}}\right|_{(i)}$$

$$[A \quad \begin{bmatrix} [\mathrm{I}] + \Delta p_{(i)}[D \quad][N_{(i)}] & -\Delta p_{(i)}[D \quad][N_{(i)}] \\ \left.\dfrac{\partial\{V\}}{\quad\{\sigma\}^{\mathrm{T}}}\right|_{(i)} & [\mathrm{I}] - \Delta p_{(i)} \left.\dfrac{\partial\{V\}}{\partial\{X\}^{\mathrm{T}}}\right|_{(i)} \end{bmatrix}$$

Calculating δΔp (correction amount of Δp)

$$\delta\Delta p = \frac{g_{1(i)} + \{m_{(i)}\}^{\mathrm{T}}\left\{\{b_{s0(i)}\} - \{b_{x0(i)}\}\right\}}{\left.\dfrac{\partial\sigma_Y}{\partial p}\right|_{(i)} \delta\Delta p - \{m_{(i)}\}^{\mathrm{T}}\left\{\{b_{sp(i)}\} - \{b_{xp(i)}\}\right\}}$$

Calculating {δσ} and {δX}
(correction amount of {σ} and {X})

$$\begin{Bmatrix} \{\delta\sigma\} \\ \{\delta X\} \end{Bmatrix} = \begin{Bmatrix} \{b_{s0(i)}\} \\ \{b_{x0(i)}\} \end{Bmatrix} + \delta\Delta p \begin{Bmatrix} \{b_{sp(i)}\} \\ \{b_{xp(i)}\} \end{Bmatrix}$$

Newton Raphson Iteration

Update

i=i+1

$$\Delta p_{(i+1)} = \Delta p_{(i)} + \delta\Delta p$$

$$\{\sigma_{(i+1)}\} = \{\sigma_{(i)}\} + \{\delta\sigma\}$$

$$\{X_{(i+1)}\} = \{X_{(i)}\} + \{\delta X\}$$

If error function is small enough?

$$g_1 \approx 0$$

$$\{g_2\} \approx \{0\}$$

$$\{g_3\} \approx \{0\}$$

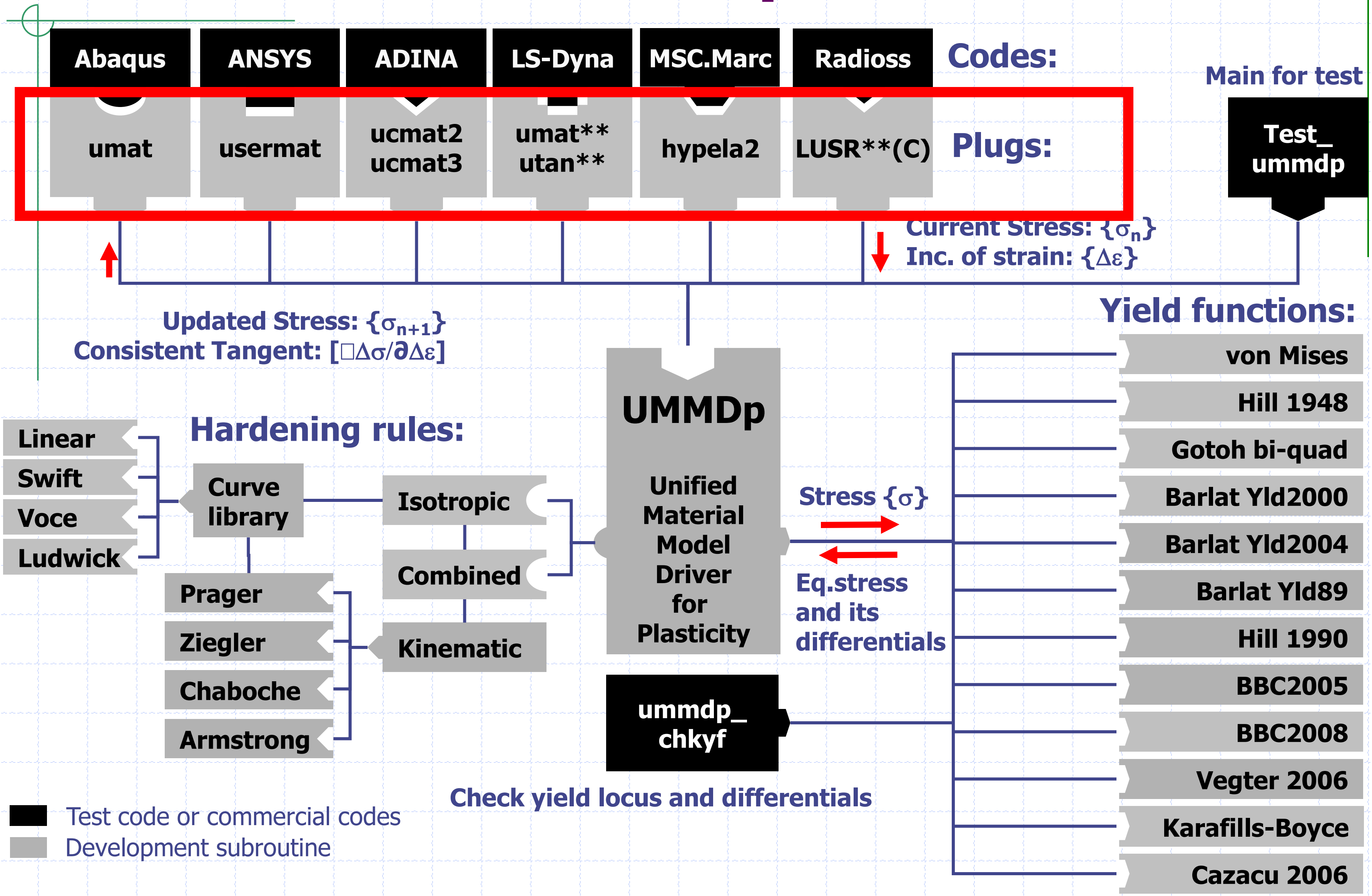
No

Yes

End of stress integration

End

Flame work of UMMDp



Codes: Abaqus, ANSYS, ADINA, LS-Dyna, MSC.Marc, Radioss

Plugs: umat, usermat, ucmat2 ucmat3, umat** utan**, hypela2, LUSR**(C)

Main for test: Test_ ummdp

Current Stress: $\{\sigma_n\}$
Inc. of strain: $\{\Delta\varepsilon\}$

Updated Stress: $\{\sigma_{n+1}\}$
Consistent Tangent: $[\square\Delta\sigma/\partial\Delta\varepsilon]$

UMMDp
Unified Material Model Driver for Plasticity

Hardening rules:

- Curve library: Linear, Swift, Voce, Ludwick
- Isotropic
- Combined
- Kinematic: Prager, Ziegler, Chaboche, Armstrong

Stress $\{\sigma\}$

Eq.stress and its differentials

Yield functions:

- von Mises
- Hill 1948
- Gotoh bi-quad
- Barlat Yld2000
- Barlat Yld2004
- Barlat Yld89
- Hill 1990
- BBC2005
- BBC2008
- Vegter 2006
- Karafills-Boyce
- Cazacu 2006

ummdp_ chkyf

Check yield locus and differentials

■ Test code or commercial codes
▒ Development subroutine

Fig.1 Frame work of development of user subroutines for UMMDp

Commercial codes

Although there are differences of variable names and storing ways, basically they have similar frameworks.

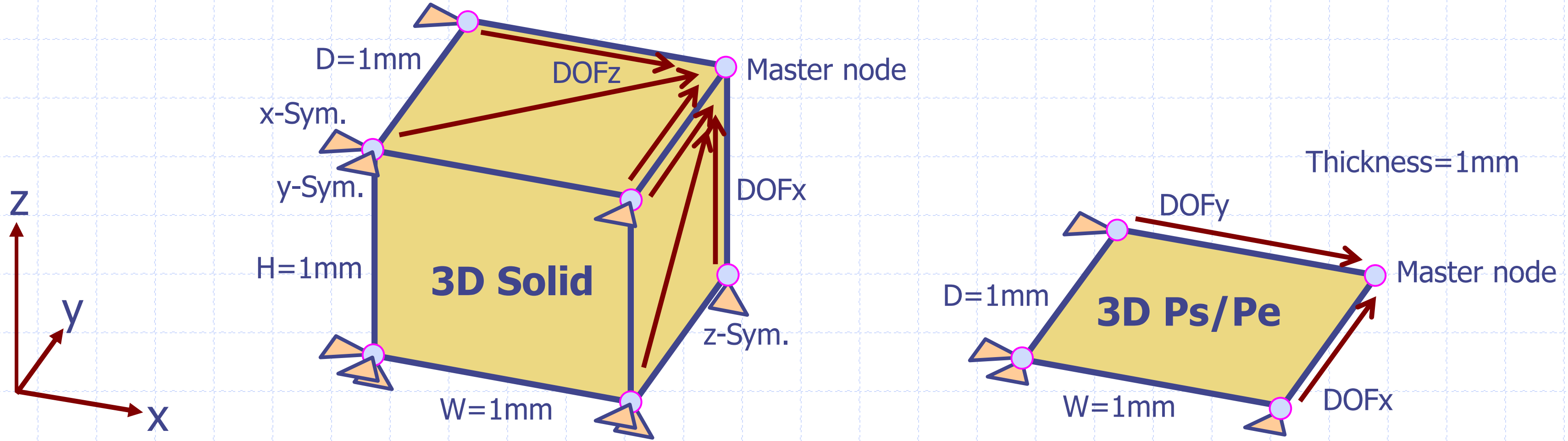
Code	Marc	Abaqus	ANSYS	LS-DYNA	ADINA	UMMDp
UsrSubR	hypela2	umat	usermat	Umat/utan	ucmat2 ucmat3	jancae_plasticity
Stress component	S	STRESS	stress	sig	stress	s1,s2
Strain increment	DE	DSTRAN	dStrain	eps	deps	de
consistent tangent modulus matrix	D	DDSDDE	dsdePI	Es	d	ddsdde
Stress component number	NGENS, NDI, NSHEAR	NTENS, NDI, NSHR	ncomp, nDirect, nShear	eltype で判断	固定 (ucmat2/3で判断)	nttl, nnrm, nshr
State variable(history variable)	T,DT	STATEV	ustatev	Hsv	ARRAY	p,dp,x(10,nttl)
Number of state variable	NSTATS	NSTATEV	nStatev	NHV	LGTH1	
End SubR	QUIT	XIT	EXIT	adios	END(KEY=4)	jancae_exit

Volunteers from software vendors checked the software own manners.

Verification Test Problems

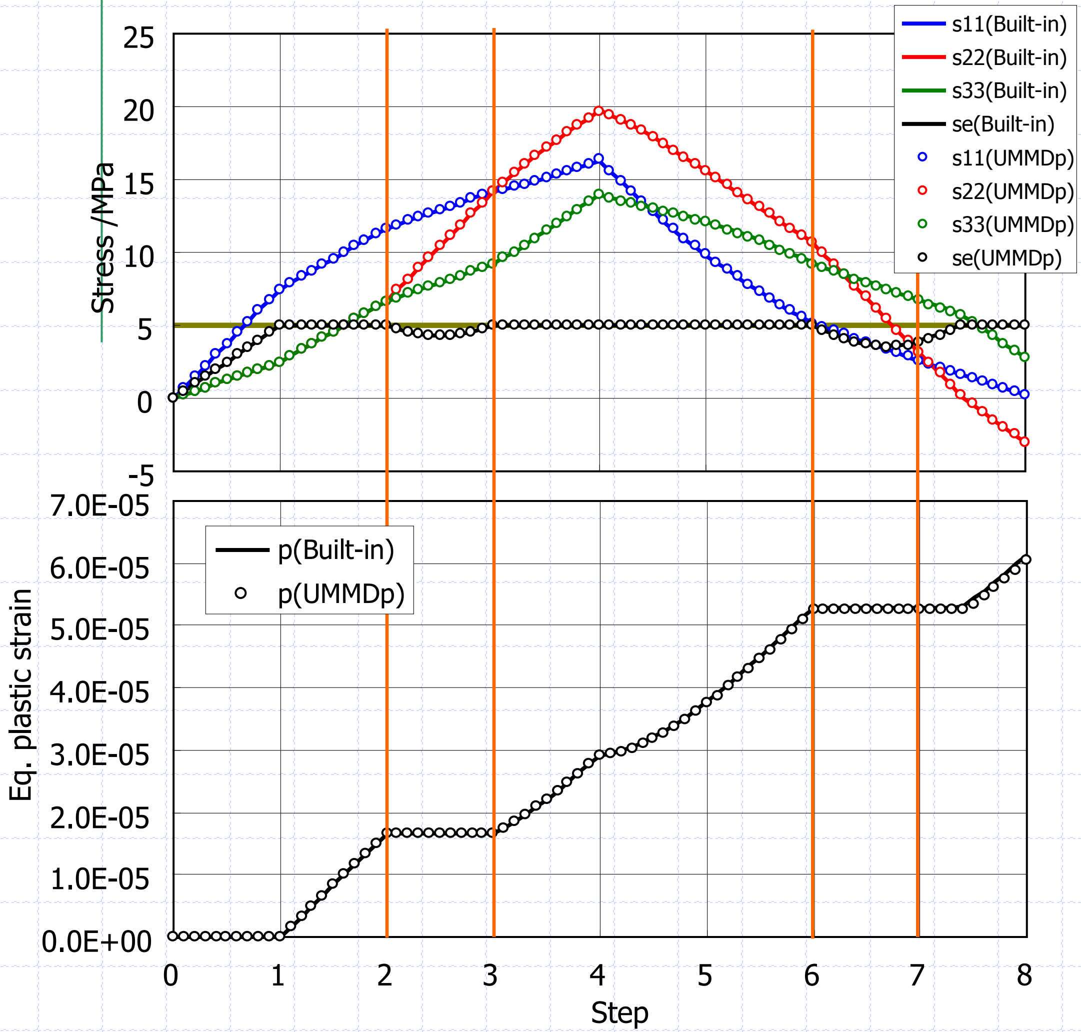
- **VT01** : Plane Strain/Disp. Control/Perfect Plasticity
- **VT02** : Plane Strain/Disp. Control/Strain Hardening
- **VT03** : Plane Stress/Disp. Control/Perfect Plasticity
- **VT04** : 3D Solid/Disp. Control/Strain Hardening
- **VT05** : 3D Solid/Force Control/Strain Hardening
- **VT06** : Plane Strain/Disp. Control/Strain Hardening (Simple Shear)

	Plane Strain	Plane Stress	3D Solid
Perfect Plasticity	VT01(Disp.)	VT03(Disp.)	--
Strain Hardening	VT02(Disp.) VT06(**Shear**)	--	VT04(Disp.) VT05(Force)

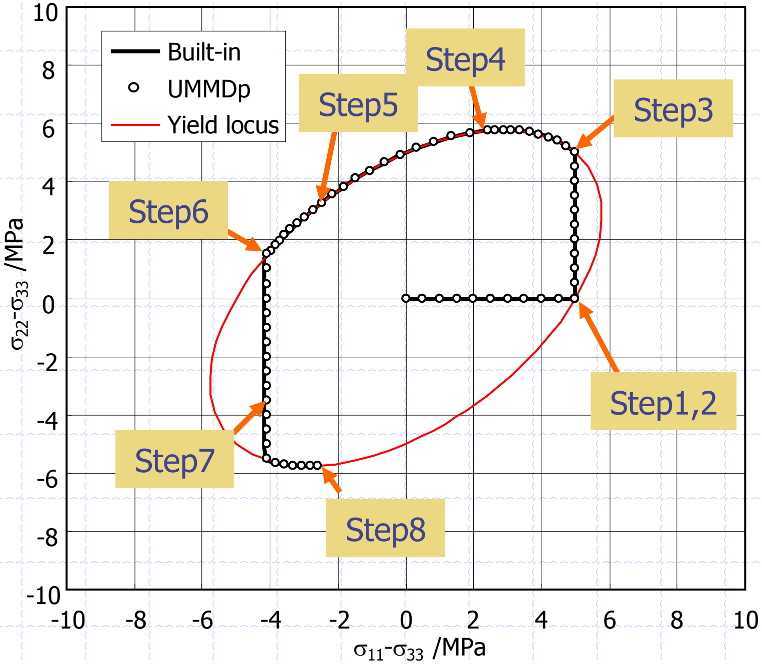


Plug for MSC.Marc

◆ Marc



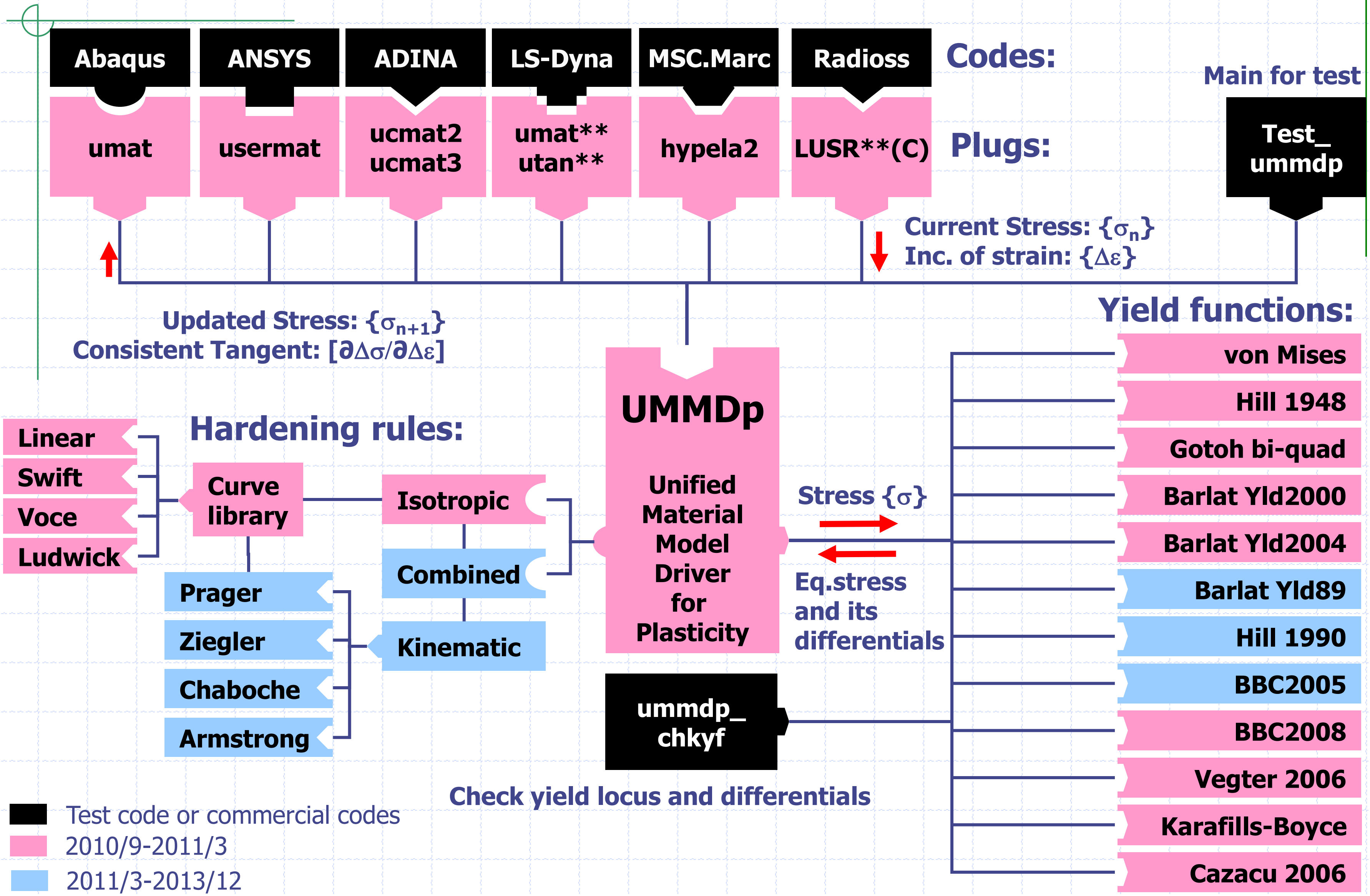
Line : Built in von Mises
Plot : UMMDp



“Plugs” by Professionals.

- Engineers form vendors
  - It was valuable to gain vendors engineers participation
    - **It’s worthy to see codes by professionals.**
  - Words from vendor engineers
    - As user subroutine is basically an exempt of user support, we don’t have an opportunity to be involved in it positively. As a result we are not necessarily understand it enough.
    - **We could establish a good relationship as numerical analysis engineers which is different from the contract based relationship.**
  - Thanks for participation
    - Participation as volunteer or self development by paying
  - What supported the motivation of vendors engineers was the effort of industrial users.

Flame work



Codes:

Abaqus **ANSYS** **ADINA** **LS-Dyna** **MSC.Marc** **Radioss**

Plugs:

umat **usermat** **ucmat2 ucmat3** **umat** utan**** **hypela2** **LUSR**(C)**

Main for test

Test_ ummdp

Current Stress: $\{\sigma_n\}$
Inc. of strain: $\{\Delta\varepsilon\}$

Updated Stress: $\{\sigma_{n+1}\}$
Consistent Tangent: $[\partial\Delta\sigma/\partial\Delta\varepsilon]$

UMMDp

Unified Material Model Driver for Plasticity

Hardening rules:

- **Linear**
- **Swift**
- **Voce**
- **Ludwick**

Curve library

Isotropic

Combined

Kinematic

- **Prager**
- **Ziegler**
- **Chaboche**
- **Armstrong**

Stress $\{\sigma\}$

Eq.stress and its differentials

ummdp_ chkyf

Check yield locus and differentials

Yield functions:

- **von Mises**
- **Hill 1948**
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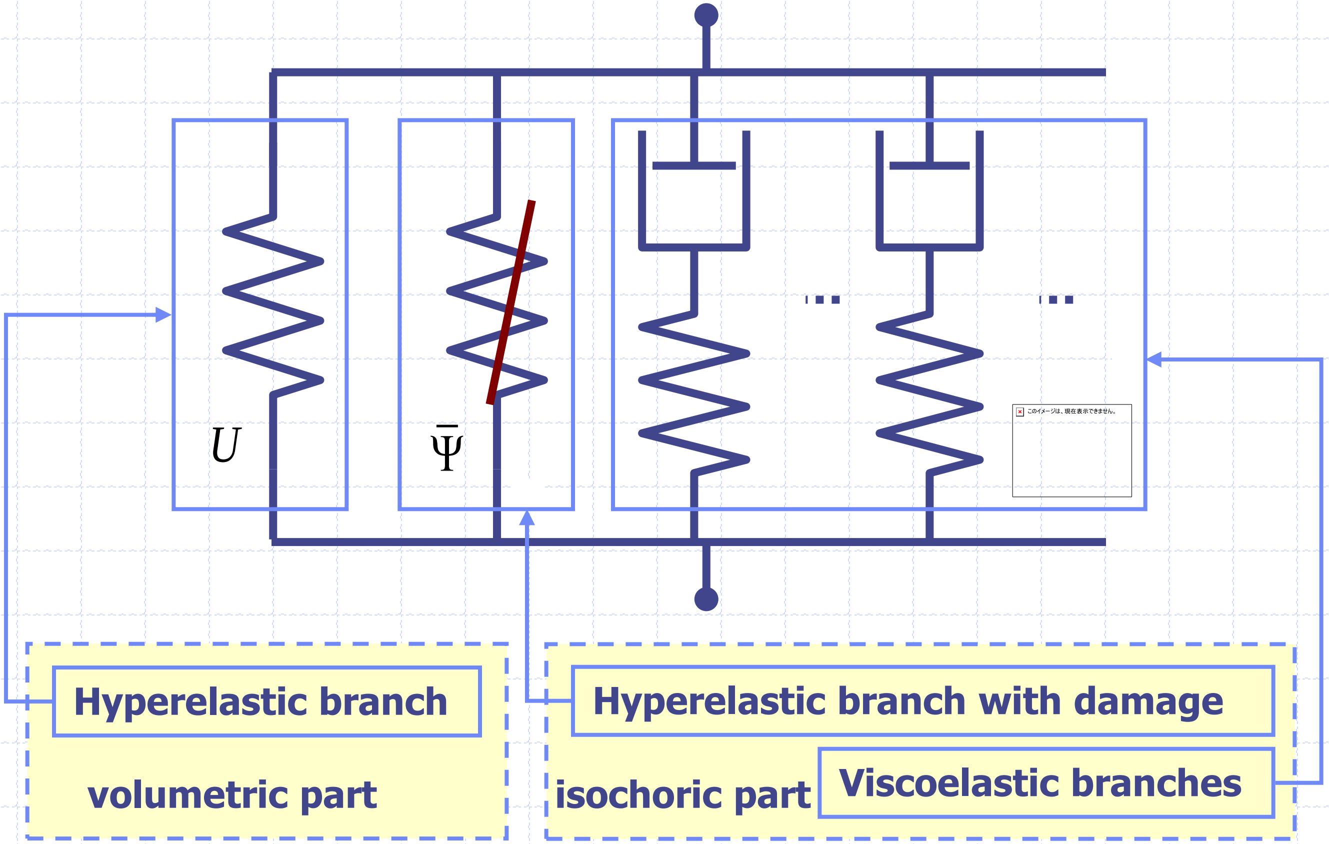
- **Hill 1990**
- **BBC2005**
- **BBC2008**
- **Vegter 2006**
- **Karafills-Boyce**
- **Cazacu 2006**

■ Test code or commercial codes
🟪 2010/9-2011/3
🟦 2011/3-2013/12

Development goes on

- Hyperelastic, viscoelastic, damage framework of constitutive law



Hyperelastic branch

volumetric part

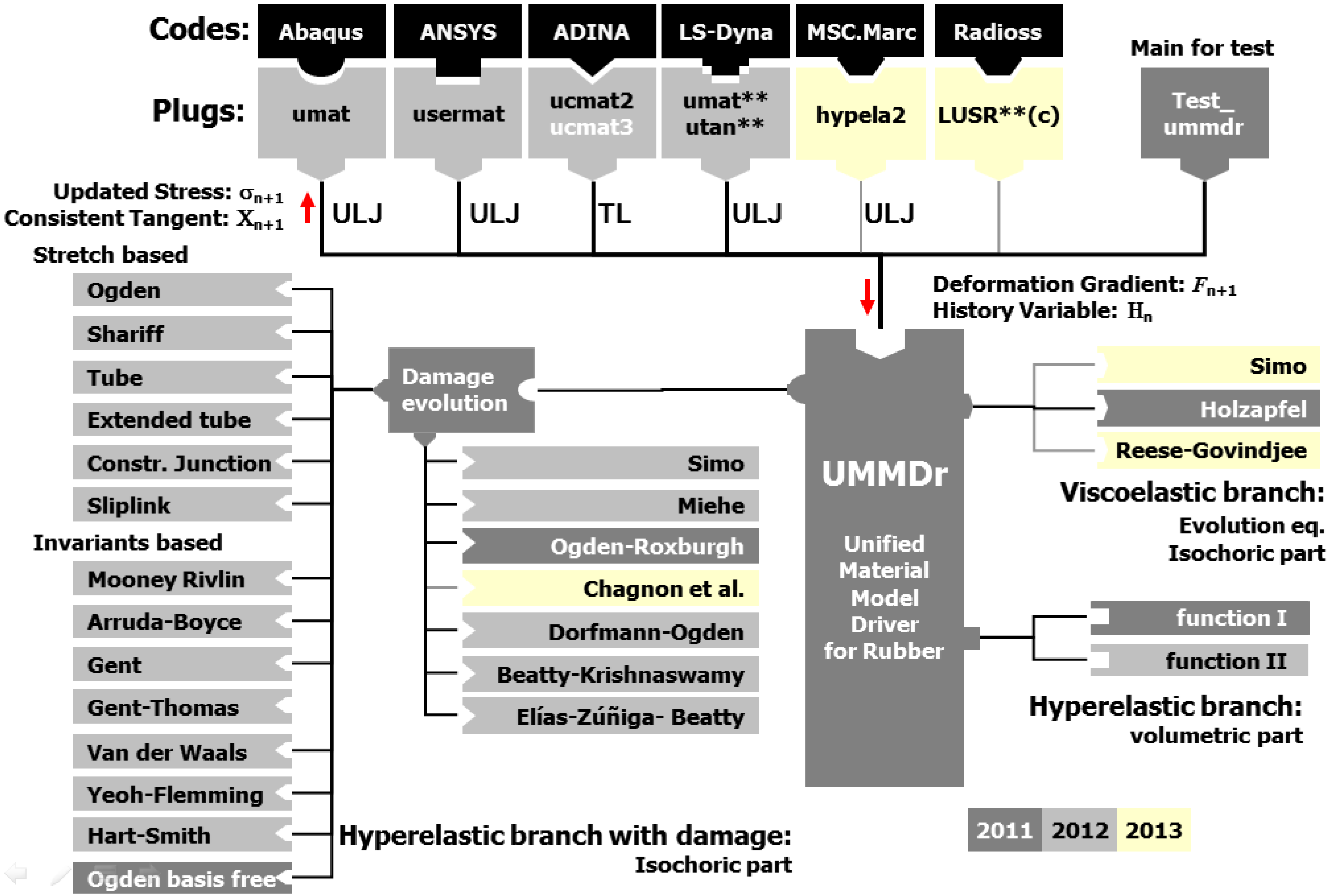
Hyperelastic branch with damage

isochoric part

Viscoelastic branches

$$\Psi(C,h,q_1,\dots,q_n) = \underset{\text{volumetric part}}{U(J)} + \underbrace{\underbrace{\bar{\Psi}(\bar{C},h)}_{\substack{\text{equilibrium part}\\\text{with damage}}} + \underbrace{\bar{\Psi}_{\mathrm{NEQ}}(\bar{C},q_1,\dots,q_n)}_{\text{non-equilibrium part}}}_{\text{isochoric part}}$$

Development goes on



Codes: Abaqus ANSYS ADINA LS-Dyna MSC.Marc Radioss

Plugs: umat usermat ucmat2 ucmat3 umat** utan** hypela2 LUSR**(c)

Main for test

Test_ ummdr

Updated Stress: σ_{n+1}

Consistent Tangent: X_{n+1}

ULJ ULJ TL ULJ ULJ

Deformation Gradient: F_{n+1}

History Variable: H_n

Stretch based

- Ogden
- Shariff
- Tube
- Extended tube
- Constr. Junction
- Sliplink

Invariants based

- Mooney Rivlin
- Arruda-Boyce
- Gent
- Gent-Thomas
- Van der Waals
- Yeoh-Flemming
- Hart-Smith
- Ogden basis free

Damage evolution

- Simo
- Miehe
- Ogden-Roxburgh
- Chagnon et al.
- Dorfmann-Ogden
- Beatty-Krishnaswamy
- Elías-Zúñiga- Beatty

Hyperelastic branch with damage:
Isochoric part

UMMDr

Unified Material Model Driver for Rubber

- Simo
- Holzapfel
- Reese-Govindjee

Viscoelastic branch:
Evolution eq.
Isochoric part

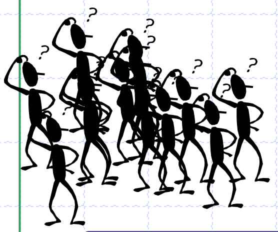
- function I
- function II

Hyperelastic branch:
volumetric part

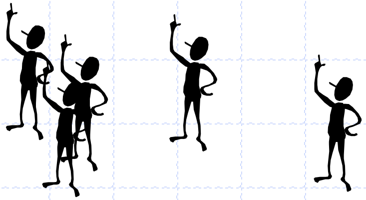
2011 2012 2013

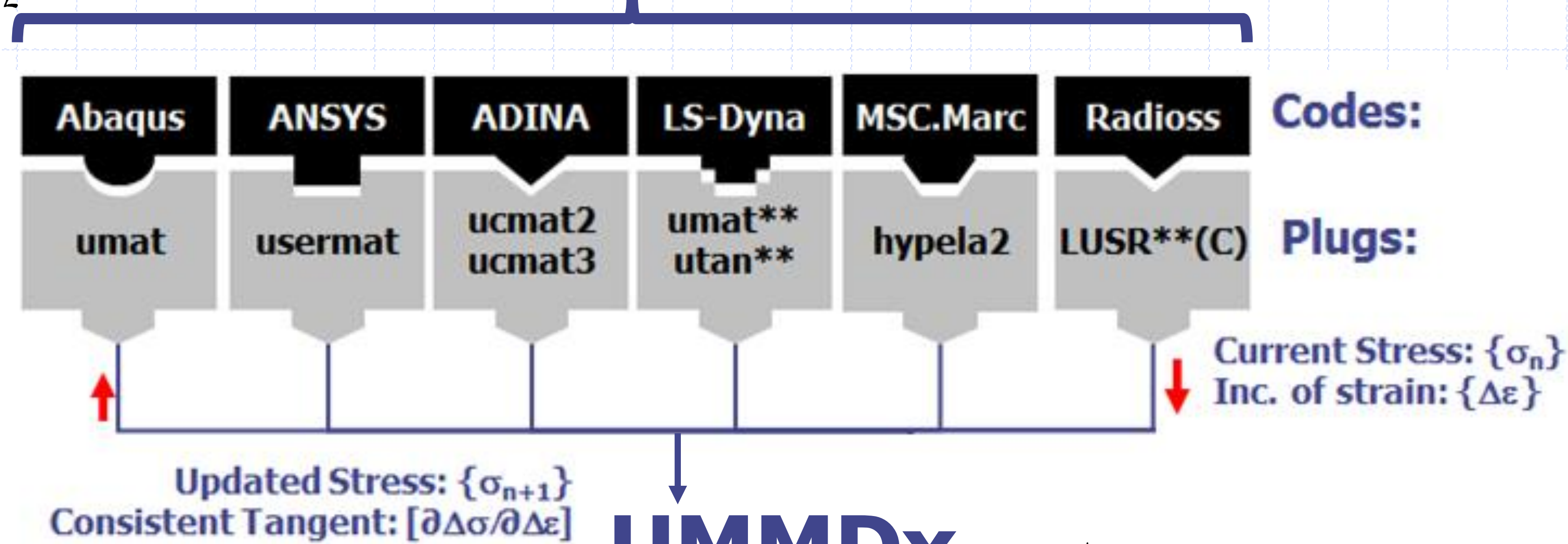
Summary(1) NPO

- Approach as NPO
  - Activities that only NPO can work on, not societies not vendors



Continuum mechanics
Numerical calculation



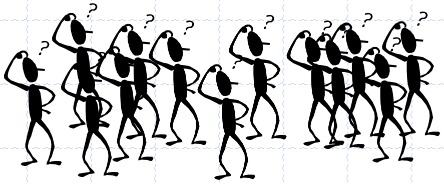


Codes:	Abaqus	ANSYS	ADINA	LS-Dyna	MSC.Marc	Radioss
Plugs:	umat	usermat	ucmat2 ucmat3	umat** utan**	hypela2	LUSR**(C)

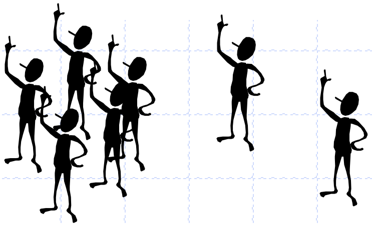
Current Stress: $\{\sigma_n\}$
Inc. of strain: $\{\Delta\varepsilon\}$

Updated Stress: $\{\sigma_{n+1}\}$
Consistent Tangent: $[\partial\Delta\sigma/\partial\Delta\varepsilon]$

UMMDx

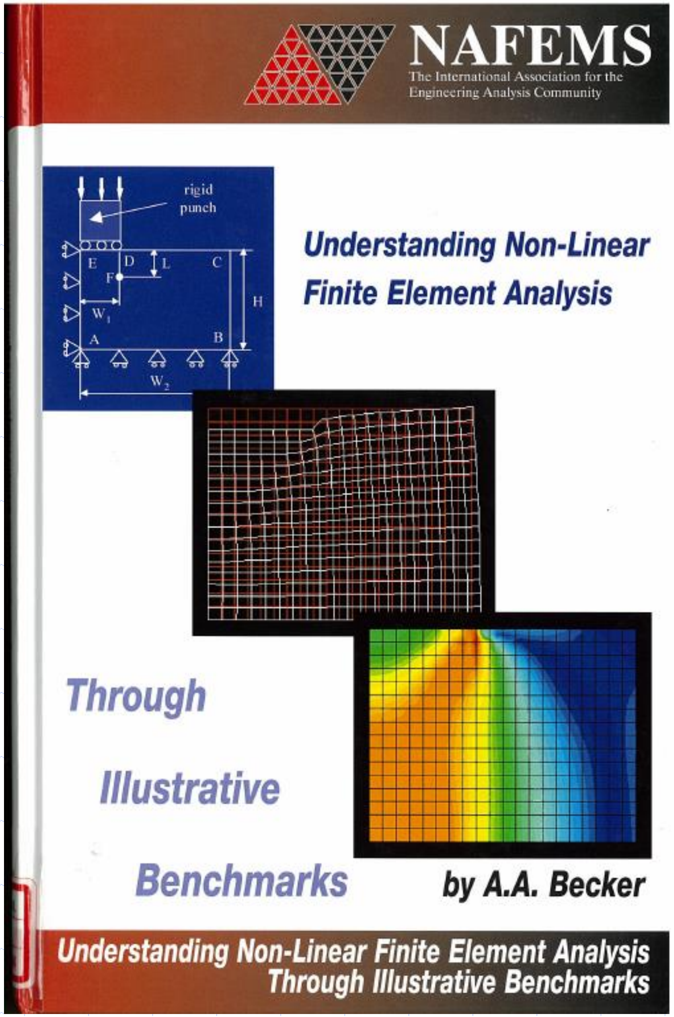


Material constitutive law



Summary(2) NAFEMS

- Verification (Benchmark)



NAFEMS

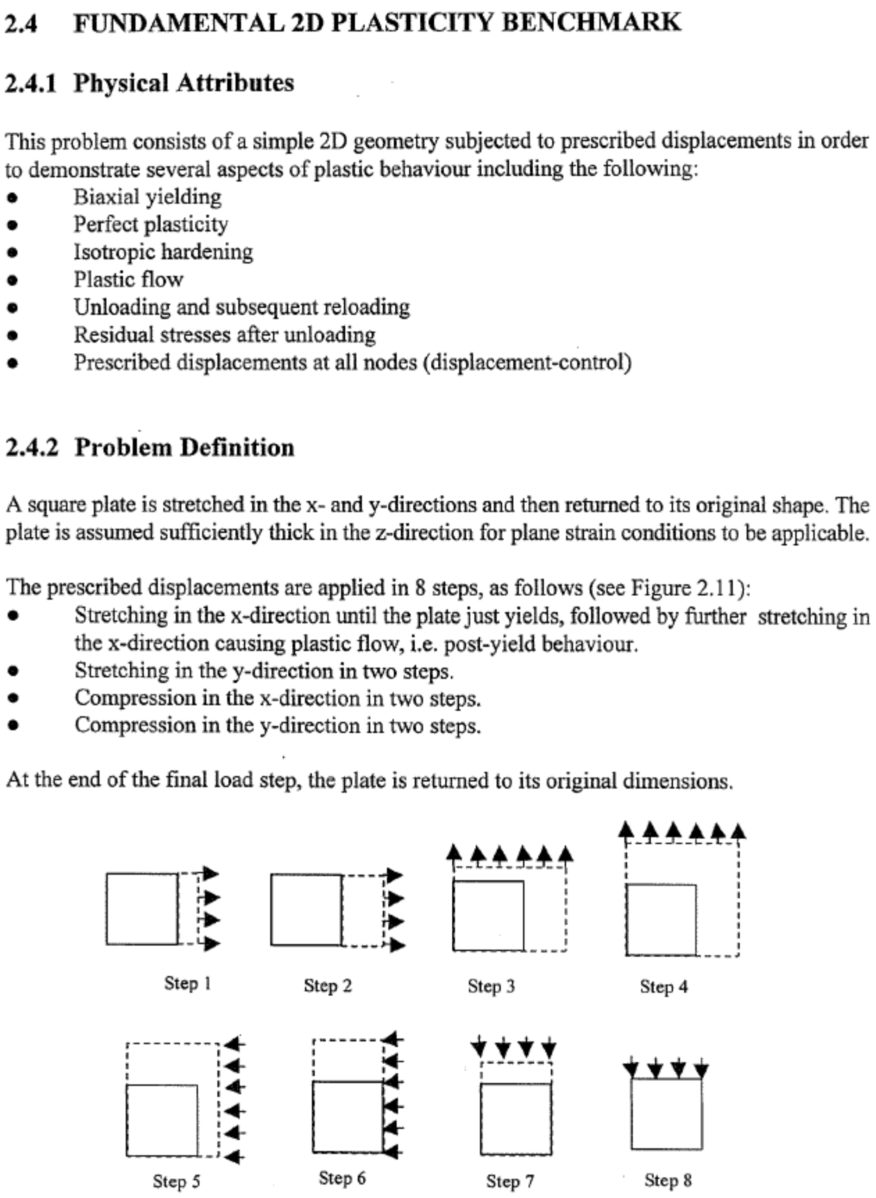
The International Association for the Engineering Analysis Community

Understanding Non-Linear Finite Element Analysis

Through Illustrative Benchmarks

by A.A. Becker

Understanding Non-Linear Finite Element Analysis Through Illustrative Benchmarks



2.4 FUNDAMENTAL 2D PLASTICITY BENCHMARK

2.4.1 Physical Attributes

This problem consists of a simple 2D geometry subjected to prescribed displacements in order to demonstrate several aspects of plastic behaviour including the following:

- Biaxial yielding
- Perfect plasticity
- Isotropic hardening
- Plastic flow
- Unloading and subsequent reloading
- Residual stresses after unloading
- Prescribed displacements at all nodes (displacement-control)

2.4.2 Problem Definition

A square plate is stretched in the x- and y-directions and then returned to its original shape. The plate is assumed sufficiently thick in the z-direction for plane strain conditions to be applicable.

The prescribed displacements are applied in 8 steps, as follows (see Figure 2.11):

- Stretching in the x-direction until the plate just yields, followed by further stretching in the x-direction causing plastic flow, i.e. post-yield behaviour.
- Stretching in the y-direction in two steps.
- Compression in the x-direction in two steps.
- Compression in the y-direction in two steps.

At the end of the final load step, the plate is returned to its original dimensions.

Summary

- Situation in Japan：
  - We are mainly using commercial CAE codes developed in other countries.
  - We focus on the applications using commercial codes.
- Convenient tools are welcome
  - The purpose of refrigerator is to keep cool.
  - The purpose of vehicle is to move safely.
  - The purpose of word processor is to layout document.
    - However word processor doesn't make text. The important of text is the what it says.
- Same for CAE. We can't use CAE without deep understanding
  - What kind of model we make for the specific problem.
  - What we understand from the result.
  - We still can't make model which perfectly recreate the "real".

Summary

- NAFEMS : Classic Documents
  - There are records of the verification for each commercial code
  - Suggesting process to accumulate our understanding not to use simulation without any consideration

- Where and how we should start
  - We can use sophisticated commercial codes if having enough budget.
  - Can we use CAE without classic records?
  - NAFEMS classic documents would be a compass of verification process for us.

- V&V : recommendation of Verification
  - Don’t we think that Verification is software vendors role and Validation is users role?
  - Validation is necessary as the common language in companies. ”**quantity**”
  - Verification is necessary to accumulate our understanding. “**quality**”