NAFEMS Benchmark Challenge

The Benchmark Challenge was launched the October 2014 edition of benchmark magazine and on the NAFEMS Challenge blog. The concept behind this initiative is to provide seemingly simply problems for readers to consider. The challenges will typically contain in interesting ‘twist’ which is intended to be thought-provoking and to provide an opportunity for learning.

The first and second challenges have received a great response. Details of these are published on the NAFEMS blog, where you can also view details of all the challenges published so far. The second challenge is still open for submissions. The overall prize of an iPad will be awarded in October, with other small prizes being given periodically.

Here, we introduce the third challenge which looks at plane strain approximations, as well as the solution to the first challenge.

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Challenge Problem 3 - The Plane Strain Approximation

Your company has recently recruited a young graduate engineer who has come to you as a seasoned professional analyst for help with understanding plane strain approximations. He had read the following sections on the derivation of plane strain equations before deciding to conduct some numerical experiments:

“Normally a plane stress approach is applied to members that are relatively thin in relation to their other dimensions, whereas plane strain methods are employed for relatively thick members.” A typical example of plane strain is the pressurisation of long cylinders where the above equations give accurate results, particularly in the middle portion of the cylinder, whether the end conditions are free, partially fixed or rigidly fixed.”
Mechanics of Materials, Volume II, 2nd Ed., E.J. Hearn, pp686-687, (1985)

The graduate engineer thought he would compare the results for a long cylinder using solid and plane strain elements. The dimensions chosen were radius 0.5m, wall thickness 0.25m and length 20m with a unit internal pressure and a material with a Poisson’s Ratio of 0.3.

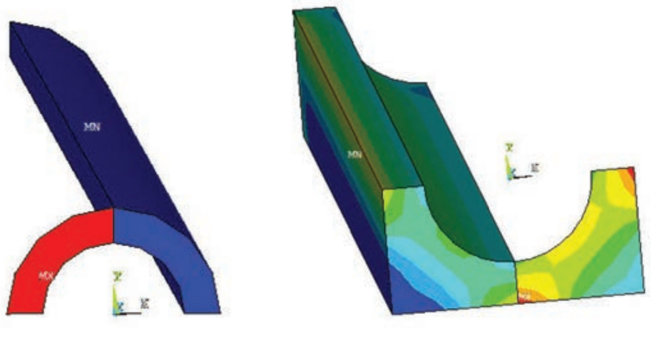

Figure 1: Szz stresses for two cylinder geometries

The solid model utilised symmetry and modelled an eighth of the cylinder using three planes of symmetry. The plane strain model also utilised symmetry with a quadrant of the cylinder modelled. The end of the solid model cylinder was left unrestrained. His main interest was to see how the models predicted the direct stress parallel to the axis of the cylinder and his plot is shown in the figure. The stress for the plane strain model is 0.48 whereas that for the solid model was effectively zero throughout the entire model (see the left hand plots in Figure 1).

He was somewhat surprised by his finding and began to consider that the text must be wrong and that for this case a plane stress assumption would provide more realistic results! He then modified the geometry by squaring off the outside of the cylinder and re-ran both solid and plane strain models. This time he did get a decent Szz variation (see the right hand plots in figure1) which was at least similar in distribution if not in magnitude with that from the plane strain model. This finding added to the confusion he felt before approaching you.

The Challenge

The challenge is to guide your young colleague towards an understanding of how plane strain assumptions work and where they might be appropriate. In doing this you might consider different geometries and different loadings and you might also consider seeing how the so-called generalised plane strain element performs. You may also wish to see what is said on this subject in other popular mechanics of material texts.

Responses should be sent to challenge@nafems.org